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OPTIMUM LOAD FREQUENCY CONTROL OF POWER SYSTEMS

by

© MOHAMED A. ABDELHALIM

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ABSTRACT

In this thesis the problem of load frequency control (LFC) of a single area and an interconnected power system is studied. The control strategy which has been chosen here adopts what is implemented in the real world of load-generation control. The control law is specified to be proportional and/or integral of the area control error (ACE). The area control error is a measure of the prevailing generation error. Different strategies have been proposed to handle the problem of LFC. The LFC problem is formulated as a parameter optimization one, in order to find the optimal control gains.

Here, the tie-line nonlinearity has been taken into account. In this case, the system nonlinearity is cast into the quadratic form which facilitates to a great extent the convergence of the numerical solution of the parametric optimization problem.

Also, the governor dead band has been taken into consideration. This problem has not been dealt with in the optimal sense until this work.

The computational aspects of the parametric optimization problem and in general the nonlinear two-point boundary value problem is discussed in this thesis.

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Chapter 1

The Role of Load Frequency Control in The Power System Operation

1.1 Introduction

The operation of an electrical power system may be viewed as a series of control actions taken to maintain continuity of service at standard frequency and voltages with minimal cost. Automation has been always integral to a power system in the form of governor action, voltage regulation and protective relaying [1]. Further automation has been achieved in the form of automatic generation control, load frequency control and economic dispatch.

In order to describe the load frequency control role in the electric power system operation, one may use the concept of multilevel control theory. Accordingly, the system mode of operation may be decomposed into the following [2]:

(1) Normal mode

(2) Preventive mode

(3) Emergency mode

(4) Restorative mode

In the normal mode of operation, the power system is operated so that the demands of all the customers are continuously satisfied at standard frequency and voltages with minimal cost. Also, the system spinning reserve should be maintained such that the system will not go unstable for minor disturbances. One can adopt a control strategy in this mode to

- keep the voltage at approximately standard voltage
- keep the frequency at standard frequency ± ϵ_f

- keep the tie-line power interchange at its scheduled value
- meet the customer demand without interruptions.

In which the frequency tolerance can be taken as

$$\varepsilon_f = 0.033 \text{ Hz}$$

according to the North American Power Systems Interconnection Committee (NAPSIC).

If a contingency is likely to occur such that the system may not be able to return to the normal mode of operation, then the system is defined to be in the preventive mode of operation. The objective of the control in this mode is

- keep the frequency at $60 \pm \varepsilon_f$
- keep the tie-line power flows at about its schedule values
- keep the customer demand at minimum cost subject to constraint on the amount of spinning reserve.

If a contingency occurs such that customer demand cannot be maintained at prespecified voltage and frequency, since otherwise the electrical system will lose synchronism, the system is in an emergency mode of operation and the control objective is

- to keep the frequency at its standard value $\pm \varepsilon_f$.
- to maximize supply of customer demand without considering economic factors.

If a contingency happens and service to some customer loads is lost, then the system is defined to be in the restorative operating mode. The control objective in this case is

- to bring the system back to the normal or preventive mode in minimum physically possible time.

It may be noted that in all modes of operation, the primary goal of the control strategy is to try to keep the frequency and tie-line power deviations within certain tolerances.

1.2 The load frequency control problem

The fact that electrical energy cannot be stored in large quantities plays an important role as far as power generation is concerned. Therefore, the mechanical output of system turbines must be continuously controlled such that the prevailing electrical generation matches exactly the consumer load demand. If a power system had the capability to do so instantaneously then the system scheduled frequency and scheduled tie-line power flow would remain constant.

Unfortunately, such a capability is practically impossible. First, exact prediction of load demand is inconveivable. Second; it is not feasible to change the generated power to match load demand instantaneously for mechanical reasons, e.g., turbine output rate of change can not exceed a certain limit due to thermal stress on the boilers. Thus, frequency and scheduled tie-line power deviations occur because of the aforementioned reasons.

1.3 The need for load frequency control

During the last three decades, the wide spread use of electric clocks has led to the need for accurate regulation of power system frequency. There are other reasons which stem from the operating principle of power systems in addition. A major reason is that there are a buy a sell power interchange agreements between adjacent areas to supply available excess lower cost energy from an area to another that can use it to replace its own higher cost energy. Exchange of power between adjacent areas of a power system is usually governed by a predetermined schedule so that during a given period of time a specified amount is exchanged. During a disturbance period, the outputs of all system generators are altered since each area of the system is involved in accommodating the load change. Consequently, the power flowing through the tie lines deviates from its scheduled values which deviates from the most economic operation mode of the system [3]. Another very important reason for keeping the frequency within a certain tolerance arises from the operating characteristic of the system turbines. When a system operates at a frequency below certain limits, some types of steam turbines undergo excessive vibrations in certain turbine rotor stages which result in a metal fatigue and blade failure. Moreover, in 60Hz systems sustained operation below 58.5Hz of such turbines may limit their operating life to one hour at full load, decreasing progressively with increasing frequency deviations [4]. When the frequency falls below approximately 58.5Hz, the turbine regulating devices will be fully open and the generating units become completely loaded. Further decrease in the frequency reduces the efficiency of the auxiliary mechanisms at steam power stations, especially feed pumps whose outputs are approximately proportional to one third of the frequency. The result of the prolonged operation under this condition is a drop in the output generation and further lows of power and increase in frequency deviation. The increase in frequency deviation may cause an avalanche of event whose nature may jeopardize the integrity of operation of the power system.

1.4 Scope of investigation

This study was motivated by efforts to design an optimal controller for the problem of load frequency control [5-28] of electric power systems.

In chapter 3, the case of designing an optimal controller for a single area, steam and hydro, power system is considered. Different optimal controllers are designed in a systematic way so as to reduce the system transients and bring the system back to its normal state.

In this case, load changes are accommodated by the system as a whole, regardless of where on the system they may occur. The control strategy is referred to as Flat-Frequency-Control. The control strategy is a proportional and/or integral of the system frequency deviation. It is shown that the optimal control gains are independent of the load variations.

The problem of LFC involves many nonlinearities in the system dynamics. The most significant of these arises from the tie-lines connecting the areas of an interconnected system, and the dead band of the area's governor. The former overides when the system is subjected to large disturbances while the latter is dominant for small system disturbances.

In chapter 4, the problem of linear and nonlinear LFC of an interconnected two area mixed power syster is studied.

On mode modern interconnected power systems, the basis for accomplishing the regulating responsibilities is the Tie-Line-Bias concept (conventional LFC) introduced 25 years ago [20]. When operating in accordance with the principle of conventional control, each area of an interconnected power system attempts to regulate its ACE to zero. This can be achieved by employing a proportional-plus-integral control of the area control error (ACE) provided that each area can fully accommodate its load changing. The ACE of an area of an interconnected power system is a measure of the mismatch of the area output generation and load demand. The problem of selecting the optimal control parameters of the conventional control strategy was studied in detail by Kirchmayer [14]. In [14], the problem of a two area interconnected power system was simulated on an analog computer. Then, by trial and error, the control parameters which introduce reasonable maximum overshoot, rise time, settling time, etc., for the frequency and tie-line power deviations were considered optimal.

Recently, several attempts have been made to employ modern control techniques to the problem of load frequency control. A significant group of results covering various approaches to design an optimal controller for the problem are available in the literature [5-20]. Elgred and Fosha [7] implemented an optimal linear regulator theory to design the supplementary regulators of a two steam area interconnected power system.

The developed control was a function of all the systems state variables and load disturbances. Therefore, it was necessary to introduce an observer in the system dynamics to realize the unmeasurable states and the load disturbances [8]. Cavin, et. al [44], considered the LFC problem from the stochastic viewpoint. In [44], the system was assumed to be subjected to a white gaussian random variable with zero mean and known covariance. The control law was also a function of all the system's state variables, however, the Kallman filter was adopted to estimate the unobservable states. The assumption that the distribution of load disturbance is white noise is far from being adequate in describing disturbances found in electric power systems. Also, the determination of a meaningful contriance matrix in practice is very difficult as detailed statistical data about a plant and its measured noise is required and such data is generally not available for a power system. Although a power system load disturbances are, in general, random by nature, it is reasonable to assume that a power system load disturbances are deterministic as modern LFC systems are designed with filters [5] in order to remove the purely random portion of the area's load disturbances leaving the deterministic components for the system units to regulate.

So for all the work mentioned before have attempted to control the frequency and the tie-line power deviations independently rather than regulating the area control error (ACE) based on the frequency bias which is the goal of tie-line-bias control used in practice. Calovic et. al., [10-11] and [51] tried to narrow the gap between the conventional and advanced LFC by letting the integral of the ACE to be a part of the control strategy [12]. In [10], the proportional part was a function of all the system state variables. In [51], both the proportional and the intergap part was a function of the output state variables. The necessary conditions to validate the developed control law depends upon the choice of a certain weighting matrix. However, as reported by the authors, there is no systematic way of computing this matrix. The economic control is a "tertiary" control, i.e., relatively slow and is based on quasi-static description of generation and demand [70]. Therefore, the economic dispatch should be treated separately from load frequency control.

In this chapter, the tie-bias-control concept which is extensively used in practice is adopted as a part of the proposed control strategy. The control law of each area of a power system is assumed to be proportional-plus-integral of the area control error (ACE). Moreover, the strategy adopts the principle that only the disturbed area supplementary regulator will react in response to its area load change (nonintervention principle); provided that the disturbed area has the capability to accommodate its load change, is adopted here for the problem of LFC of interconnected power system.

Here the problem is formulated as a parameter optimization one. It has been found that the optimal control parameters are independent of the load variations in contrast to [7], and therefore there is no need to identify it [8]. Moreover, the proposed strategy adopts the principle that only the disturbed area supplementary regulator will react in response to its area load change (non-intervention principle); provided that the disturbed area has the capability to accommodate its load change. This is in contrast to most of the recent proposed strategies [5-28].

Here, the problem of LFC of interconnected power system; either linear or nonlinear, is formulated as a parameter optimization one. In the case of the linear one, the optimal control parameters have been found to be independent of the load variations and therefore there is no need for incorporating an identifier (observer) in the system dynamics to estimate the load demand [8].

Many efforts have been done to study the nonlinear version of the LFC problem; tie-line nonlinearity. Miniesy and Bohn [16] have proposed a two level controller for the problem of nonlinear load frequency control. The first level is a local feedback control for each area of the interconnected system. This control is a function of its own area state variables. The second level is an intervention open loop which is utilized to compensate for neglecting the coupling state variable between the areas of the interconnecting system, and the nonlinearities due to the tie-lines. The idea of employing a multi-level control strategy in the area of load frequency control is not adequate. First, it increases the controller design complexity.

Second, it is not used in practice. Doraiswami [17] has considered the problem of LFC with a tie-line nonlinearity from the stochastic viewpoint. An observer was employed to implement the control law.

Here, the system nonlinearity is cast into the quadratic form and the problem is formulated as a parameter optimization one. The optimal control parameters have been found to be dependent on the load variations. However, it is shown that the suboptimal control parameters obtained from the linear model, are good enough to design supplementary area regulators with insignificant (in a practical sense) degradation of system performance. This eliminates the necessity of designing an adaptive controller [17] and therefore simplifies the design.

In chapter 5, the governor deadbank of a single area steam power system is studied.

With the recognition that a well-developed linear control theory now exists, more research is being directed towards nonlinear aspects of general control systems. Linear control theory (classical or optimal) suffers from the fundamental criticism that, in reality, dynamical systems are frequently subject to several complicating factors which may invalidate or at least severely limit its applicability. There are no general methods for analysis and synthesis of nonlinear control systems in classical or modern control theory. For example, dealing with systems with control signal saturation (relay control systems [71] and [72] employing modern control theory is quite different from dealing with systems with, e.g., output feedback which contains a functional nonlinear element [73].

Here we deal with a system whose dynamics comprise a hysteresis element. The approach developed here (chapter 5) is general and is applied to the problem of finding the optimal control parameters of the supplementary regulator of single area steam power systems.

The problem of finding the optimal control parameters has been reduced to the problem of solving a nonlinear two-point-boundary-value problem (TPBVP) with multiple corners. This has been handled by employing the Weierstrass-Erdmann conditions for optimality [29]. A technique is developed to detect when a corner and hence the problem can be treated as ordinary TPBVP.

Chapter 2

Parametric Optimization

2.1 Introduction

Parametric optimization techniques represent a wide class of aids in designing control systems. The tools which are required to perform parameter optimization processes depend, among other things, upon the nature of the input signal to the dynamic system under consideration. The nature of the input signal to any dynamic system is either stochastic or deterministic processes. In this dissertation, systems subjected to deterministic inputs are considered.

2.2 The problem of deterministic parameter optimization

The parameter optimization problem considered here may be formulated in terms of ordinary differential equations of the form

$$\dot{X}(t) = f(X(t),C), \quad t\epsilon[t_o, t_f] \qquad (2.1)$$

Here, t is the independent variable, X is an n-dimensional vector whose components are the dependent variables, anc C is an m-dimensional vector whose components represent the salient variables of the system. C-vector is assumed to be time-independent, that is,

$$\dot{C} = 0, \qquad (2.2)$$

When the parameters in C and the complete initial conditions

$$X(t_0) = X_0 \qquad (2.3)$$

are known, a numerical integration of Eqn. (2.1) produces the solution X(t) on the time interval $t_0 \leq t \leq t_f$. This initial value problem can readily be solved with a digital or analog computer.

On the other hand, the parameter optimization problem is the problem of finding the parameter vector C which maximizes or minimizes some scalar performance index

$$J = \int_{t_0}^{t_f} \Phi[X(t), C, t]dt + G[X(t_f), C] \qquad (2.4)$$

subject to system dynamic constraints given by Eqns. (2.1) - (2.2). It is assumed that the integrand Φ has continuous first partial derivatives with respect to all of its arguments; t_0 and t_f are fixed. It is also assumed that the trajectory of X(t) have only piecewise-continuous first derivative; that is, $\dot{X}(t)$ will be continuous except at a finite number of times in the interval (t_0, t_f).

It can be seen that Eqn. (2.4) is sufficiently general to investigate a wide class of practical problems.

Only in some simple parameteric optimization cases is it possible to obtain an analytic solution, but in most cases a numerical method must be used. There are many numerical methods available for this purpose, e.g. [30-32]; some of these references require the evaluation of the performance index only, while others require the evaluation of the gradient, and possibly, of the higher order derivatives of the performance index.

The method of gradients which has been applied with considerable successes to a variety of problems, is adopted here to solve the forementioned parametric optimization problem. The method of gradient requires

1 - The evaluation of the performance index, and

2 - The evaluation of the gradient of the performance index with respect to the parameters.

The first requirement can be carried out by using a numerical integration process by augmenting the system dynamic equations, Eq. (2.1), by adding the equation

$$\dot{x}_0 = \Phi[X(t),C,t], \qquad x_0(t_0) = 0 \tag{2.5}$$

Then, Eqns. (2.1) and (2.4) are evaluated in the forward-time to yield the required value of the performance index

$$J = x_0(t_f) + G[X(t_f)C] \tag{2.6}$$

The second requirement, the evaluation of the gradient of the performnce index,

$$J_C = \frac{\partial J}{\partial C} \tag{2.7}$$

can be carried out conveniently in terms of a Hamiltonian defined by

$$H = \Phi + \Lambda^T f \tag{2.8}$$

In Eqn. (2.8), Λ is referred to as the co-state vector, and, is defined by

$$\Lambda = -\frac{\partial H}{\partial X}, \quad \Lambda(t_f) = \frac{\partial G}{\partial X}\Big|_{t_f} = G_{x_f} \tag{2.9}$$

Then the gradient of the performance index can be shown to be given by the following

$$J_C = \int_{t_0}^{t_f} H_C \, dt + G_C \tag{2.10}$$

where

$$H_C = \frac{\partial H}{\partial C} \tag{2.11}$$

$$G_C = \frac{\partial G}{\partial C} \Big|_{t_f} \tag{2.12}$$

and it can be evaluated conveniently by introducing the new variables

$$\dot{Q} = - H_C, \quad Q(t_f) = G_C \tag{2.13}$$

Then, the integrating of Eqns. (2.9) and (2.13) backward in time, yield

$$J_C = Q(t_0) \tag{2.14}$$

provided that the state vector from the forward-time integration has been stored. The scheme of forward-time and backward-time integration used here is typical of the two-point boundary value problems, and generally arises in the optimization problems defined on finite time intervals.

The two-point boundary value problem arising in the parametric optimization problem considered here can be described as follows:

(i) Forward-time equations:

$$\begin{aligned} \dot{X} &= f[X(t), C], & X(t_0) &= X_0 \\ \dot{x}_0 &= \Phi[X(t), C, t], & x(t_0) &= 0 \end{aligned} \tag{2.15}$$

The value of the performance index

$$J = x_0(t_f) + G[X(t_f), C] \tag{2.16}$$

can be found by solving equations (2.15) forward in time.

(ii) Backward-time equations:

$$\begin{aligned} \dot{\Lambda} &= -H_X, & \Lambda(t_f) &= G_{x_f} \\ \dot{Q} &= -H_C, & Q(t_f) &= G_C \end{aligned} \tag{2.17}$$

The value of the gradient of the performance index with respect to C

$$J_C = Q(t_0) \tag{2.18}$$

can be found by solving equations (2.17) backward in time.

Once the gradient of the performance index with respect to the vector C is found, then one can solve the two-point boundary value problem described by Eqns. (2.15) and (2.17), by using successively the expression

$$C = \alpha D + E \tag{2.19}$$

where D is a feasible direction depends on J_E, α is an optimal or suboptimal step size, and E is the previous estimate of the vector C.

One of the well known methods to determine a feasible direction D is the "Method of steepest descent" or the "method of gradient". Recall that the direction of steepest ascent of J at a given value E of C is equal to J_E. The vector $- J_E$ therefore points in the direction of steepest descent. Starting with an estimate parameter vector E we minimize J successively along lines in directions of the steepest descent. The method of gradient is useful for a large class of well conditioned problems. However, experience has shown that the method can be extremely slow [33]. Fortunately, there is a nice modification of the gradient method, called the conjugate-gradient method, which yields the optimum solution much faster than the method of steepest descent. In our work here, the "method of conjugate-gradient is employed to find the direction D appearing in Eqn. (2.19).

Further, we determine the optimal step size α by solving the one-dimensional minimization problem

$$\min_{a > 0} J(E + \alpha D) \tag{2.20}$$

2.3 Conjugate-gradient algorithm

The method of conjugate-gradient is a special case of the method of conjugate directions where the successive feasible directions are related to each other by the relationships

$$D_i^T \; Q \; D_j = 0 \; , \quad i,j = 1,2,\ldots, \; i \neq j \tag{2.21}$$

where Q is any positive definite matrix. In this case we say that D_i and D_j are Q-conjugate. The method of conjugate-gradient associates conjuancy properties, Eqn. (2.21), with the method of steepest descent in one algorithm in order to achieve both efficency and reliability. In the case of optimizing a quadractic performance index, the method of conjugate-gradient is an exact one-dimensional search method [33]; i.e. it does not require a one-dimensional search in conjunction with the gradient-conjugate algorithm. Note that, this method is not an exact one-dimensional search method for a nonquadratic performance index.

In the case of nonquadratic performance index, there are a number of algorithms that might be adopted in conjunction with the conjugate-gradient algorithm. Since the gradient of the performance index has to be found to determine the feasible directions, it is reasonable to use a version of a one-dimensional search algorithm based on the derivatives of the performance index.

Here, the restrictive extrapolation and interpolation technique developed by Fletcher-Reeves [33] is employed to find the optimal step size α.

Let vector G be defined as the performance index gradient with respect to C; that is,

$$G_i = J_C^{\,i} \tag{2.22}$$

Then the feasibility direction vectors can be expressed as

$$D_i = G_i + (G_i^T G_i / G_{i-1}^T G_{i-1}) D_{i-1} \tag{2.23}$$

We shall refer each computation for a feasible direction and corresponding optimization processes in one-dimension as an iteration.

Further each minimization of $J(C + \alpha D)$ with respect to $\alpha > 0$ will be referred to as a trial. The general trial procedure can be specialized by selecting initialization, extrapolation, and interpolation procedures.

The basic steps of finding the optimal parameters by the method of conjugate-gradient are:

(1) Select a reasonable value of the parameter C^i, and store it in the memory of the digital computer. Let the iteration index i be zero.

(2) Using the nominal parameter C^i, integrate the state equations (2.8) from t_0 to t_f with initial conditions $X(t_0) = X_0$ and store the resulting trajectory $X^{(i)}$ as a piecewise-constant vector function.

(3) Calculate $\Lambda^i(t_f)$ and $Q^i(t_f)$. Then, integrate equations (2.15) backward in time from t_f to t_0 using the piecewise-constant vector stored in step (2). Then, evaluate the performance index gradient with respect to parameter vector from equation (2.16) and store the vector J_C.

(4) If

$$||J_{C^i}|| \leq \epsilon_1$$

where ϵ_1 is a prespecified positive constant, and

$$||J_{C^i}||^2 = \int_{t_0}^{t_f} J^T_{C^i}\, dt \qquad (2.23)$$

terminate the iterative procedure. If the iterative stopping criterion, equation (2.22) is not satisfied, perform the trial process:

a - Select a reasonable initial value of the step size α_0. Here, quadratic extrapolation based on an estimate of the minimum value of the performance index; J_{min}, or a prespecified α_{min}, results in the initialization procedure [32]. Take $\alpha_0 = 0$, then calculate

$$\alpha_1 = \min\{2 \frac{J_{min} - J(C^i)}{D^T G_i}, \alpha_{min}\} \qquad (2.24)$$

b - Using the step size as in equation (2.24), calculate $J_{C^i+\alpha d}$. This is done through steps (2) and (3).

c - If

$$\left| \frac{D^T G_{C^i+\alpha_1 D_i}}{D^T G_{C^i}} \right| \leq \varepsilon_2 \qquad (2.25)$$

where ε_2 is a given positive constant < 1 terminate the trial process and return to step (2). If the trial stopping criterion, equation (2.25), is not satisfied, a restrictive extrapolation or interpolation is implemented to generate a new step size.

$$\alpha = \alpha_0 + (\alpha_1 - \alpha_0) r \qquad (2.26)$$

In the case of $J_\alpha(\alpha_0) < J_\alpha(\alpha_1) \leq 0$, the performance index $J(\alpha)$ is approximated by a quadratic function $H(\alpha)$, of the form [71]

$$H(\alpha) = a + b\alpha + c\alpha^2 \qquad (2.27)$$

If the values of $J(\alpha_0)$, $J_\alpha(\alpha_0)$, and $J_\alpha(\alpha_1)$ are available, then the parameters a, b, and c of H can be determined by solving the equations

$$a + b\,\alpha_0 + c\,\alpha_0^2 = J(\alpha_0)$$

$$b + 2\,c\,\alpha_0 = J_\alpha(\alpha_0) \qquad (2.28)$$

$$b + 2\,c\,\alpha_1 = J_\alpha(\alpha_1)$$

The extrapolating quadratic polynomial can be written accordingly by

$$H(\alpha) = J(\alpha_0) + J_\alpha(\alpha_0)[\alpha-\alpha_0] +$$

$$0.5\,(\alpha-\alpha_0)^2\,\frac{[J_\alpha(\alpha_0) - J_\alpha(\alpha_1)]}{\alpha_0 - \alpha_1} \qquad (2.29)$$

Condition that $H_\alpha(\alpha) = 0$, yields

$$\alpha = \alpha_0 + (\alpha_1 - \alpha_0)r \qquad (2.30)$$

where

$$r = \frac{J_\alpha(\alpha_0)}{J_\alpha(\alpha_0) - J_\alpha(\alpha_1)} \qquad (2.31)$$

However, if $J_\alpha(\alpha_0) \geq J_\alpha(\alpha_1)$ then α does not exist, and if $J_\alpha(\alpha_0) - J_\alpha(\alpha_1)$ is small, then α may be unacceptably large. For these reasons we restrict α so that in Eqn. (2.26) the quantity r will be in the form

$$r = \begin{cases} \dfrac{J_\alpha(\alpha_0)}{J_\alpha(\alpha_0) - J_\alpha(\alpha_1)} & \text{for } J_\alpha(\alpha_1) \geq (1 - \dfrac{1}{r_{max}})\,J_\alpha(\alpha_0) \\ r_{max} & \text{for } J_\alpha(\alpha_1) < (1 - \dfrac{1}{r_{max}})\,J_\alpha(\alpha_0) \end{cases} \qquad (2.32)$$

and

$$r\varepsilon(1,\ r_{max}), \qquad (2.33)$$

where r_{max} is chosen as follows

$$r_{max}\ \varepsilon(5,\ 10). \qquad (2.34)$$

In the case of the conditions that $J_\alpha(\alpha_0) < 0$ and $J_\alpha(\alpha_1) > 0$ are satisfied a cubic interpolation can be implemented to find the optimal step size. As before $J(c + \alpha d) = J(\alpha)$ can be approximated by a cubic function $H(\alpha)$ [71] in the form,

$$H(\alpha) = a + b\alpha + c\alpha^2 + d\alpha^3 \qquad (2.35)$$

Assuming that the values of $J(\alpha_0)$, $J_\alpha(\alpha_0)$, $J(\alpha_1)$, and $J_\alpha(\alpha_1)$ are available, then the parameters of H can be determined from the equations

$$\begin{aligned}
a + bJ(\alpha_0) + c[J(\alpha_0)]^2 + d[J(\alpha_0)]^3 &= J(\alpha_0) \\
a + bJ(\alpha_1 + c[J(\alpha_1)]^2 + d[J(\alpha_1)]^3 &= J(\alpha_1) \qquad (2.36) \\
b + 2c\ J(\alpha_0) + 3d[J(\alpha_0)]^2 &= J_\alpha(\alpha_0) \\
b + 2c\ J(\alpha_1) + 3d[J(\alpha_1)]^2 &= J_\alpha(\alpha_1)
\end{aligned}$$

In this case, the interpolating polynomial $H(\alpha)$ is of the form

$$H(\alpha) = J(\alpha_0) + J_\alpha(\alpha_0)(\alpha_1-\alpha_0) - [Z + J_\alpha(\alpha_0)](\alpha-\alpha_0)$$

$$+ \frac{1}{3}[J_\alpha(\alpha_0) + J_\alpha(\alpha) + 2Z](\alpha-\alpha_0) \tag{2.37}$$

$H_\alpha(\alpha) = 0$ yields

$$\alpha = \alpha_0 + (\alpha_1 - \alpha_0)r \tag{2.38}$$

where

$$r = \frac{J_\alpha(\alpha_0) + Z + q}{J_\alpha(\alpha_0) + J_\alpha(\alpha_1) + 2Z} \tag{2.39}$$

in which

$$Z = 3\,\frac{J(\alpha_0) - J(\alpha_1)}{(\alpha_1 - \alpha_0)} + J_\alpha(\alpha_0) + J_\alpha(\alpha_1) \tag{2.40}$$

$$q = [Z^2 - J_\alpha(\alpha_0)\,J_\alpha(\alpha_1)]^{1/2} \tag{2.41}$$

when

$$J_\alpha(\alpha_0) + J_\alpha(\alpha_1) + 2Z = 0 \tag{2.42}$$

then, r is given by

$$r = \frac{J_\alpha(\alpha_0)}{J_\alpha(\alpha_0) - J_\alpha(\alpha_1)} \tag{2.43}$$

Therefore, condition $H_\alpha(\alpha) = 0$ yields an estimate α in the form of (2.20) with r as follows

$$r = \begin{cases} \dfrac{J_\alpha(\alpha_0) + z + q}{J_\alpha(\alpha_0) + J_\alpha(\alpha_1) + 2z} & \text{for } J_\alpha(\alpha_0) + J_\alpha(\alpha_1) + 2z \neq 0 \\[2ex] \dfrac{J_\alpha(\alpha_0)}{J_\alpha(\alpha_0) - J_\alpha(\alpha_1)} & \text{for } J_\alpha(\alpha_0 + J_\alpha(\alpha_1) + 2z = 0 \end{cases} \qquad (2.44)$$

d - Replace α_0 by α_1 and α_1 by α and return to step b in the trial process.

2.4 Integration methods

It has been shown here, that each computation of J(C) and J_C requires the computation of the state and co-state equations. For an efficient computational procedure, then, it is desirable to minimize the time required for the integration process. This reduction can be achieved by employing a predictor-corrector type of integration scheme.

Predictor-corrector methods require the computation of the derivatives two times in each integration step, as compared to four times for the standard Rune-Kutta method [34]. Here Hamming predictor-corrector scheme has been employed to perform the required integration processes in section (2.3).

2.5 Remarks

It is obvious that the success of the gradient methods of optimization is somewhat dependent on judgement and intuition of the user. It has been successfully applied to some practical problems such as the problem of load frequency control which is the subject of this dissertation.

A computer program has been written in Fortran IV to simulate the proceeding algorithm. A complete list of the code is given in Appendix A. This code can be used for large classes of optimal control design problems.

Chapter 3

Load Frequency Control of Single Area Systems

3.1 Introduction

A single area system is one in which load changes are accommodated by the system as a whole, regardless of where they occur on the system. Load changes that occur in any part of the system may be absorbed elsewhere within the system, in accordance with allocation practices prevailing at that particular time. No one part of the system is expected to adjust its own generation to counteract its own load changes.

Most large power systems are geographically wide-spread, made up of different operating companies. Experience in automatic operation of these systems sometimes shows a need to assign regulation of system frequency to one of these companies in the group centrally located with respect to other companies. Regulation of the remaining companies in the system depends on the frequency and the power flow of the tie-lines connecting the respective companies to the central area. Therefore, the central area tie-line power flows are neither scheduled nor controlled. A load frequency control which adopts complete relaxation of the tie-line power interchange between the companies of a system is referred to as Flat-Frequency Control.

This chapter is devoted to the study of load frequency control for systems or areas which employ a flat frequency control strategy. The parameter optimization technique developed in chapter 2 is employed to design an optimal controller for the frequency control of electric power generation.

3.2 Load frequency control of single area steam power system

3.2.1 Problem formulation

A typical linearized model of a single area steam power system; SASPS, is shown in fig. (1) [15]. On state variable form, the system dynamics can be written as follows:

$$\dot{X}_1 = A_1X_1 + B_1\, u + \Gamma_1\, \Delta_1 \qquad (3.1)$$

$$Y_1 = C_1X_1$$

where

$$X_1^T = (\Delta\omega,\ \Delta x_g,\ \Delta p_g)$$

in which

$\Delta\omega$ = angular frequency deviation rad/sec.

Δx_g = deviation in governor position in pu power

Δp_g = deviation in turbine output power in pu power

u = speed changer position in pu power

ΔL = step load change

and

$$A_1 = \begin{array}{|c|c|c|} \hline -\frac{G}{M} & 0 & \frac{1}{M} \\ \hline -\frac{E}{T_g} & -\frac{1}{T_g} & 0 \\ \hline & \frac{1}{T_t} & -\frac{1}{T_t} \\ \hline \end{array}$$

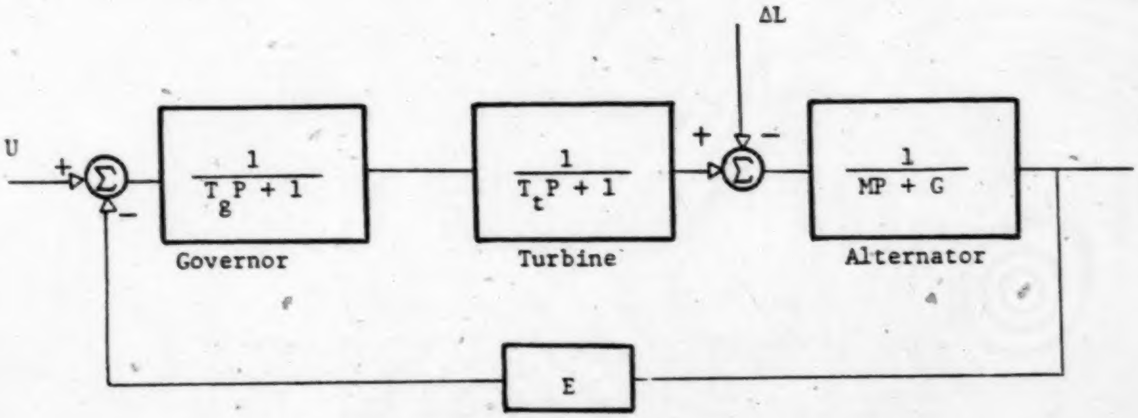

Fig. (1) Block diagram of a single steam area power system

$C_1 = (\ 1\ 0\ 0\ )$, $B_1 = (1, 0, 0)$, and $\Gamma_1 = (1, 0, 0)$

The control is assumed to be integral or proportional -plus-integral of the angular frequency deviation. In terms of the system state variables, the control law can be written as follows:

$$u = - k_p x_1 - k_I \int_{t_0}^{t_f} x_1 dt \qquad (3.2)$$

in which

k_p = proportional gain
k_I = integral gain

the control parameters k_p and k_I can be expressed mathematically as follows:

$$\dot{K} = 0 \qquad (3.3)$$

where

$K^T = (k_p, k_I)$

The problem posed is to find the optimal control u or equivalently the optimal integral gain k_I and/or the proportional-plus-integral control parameters k_p and k_I which minimize the cost functional

$$J_1 = \frac{1}{2} \int_{t_0}^{t_f} (X_1^T Q_1 X_1 + \gamma u^2) dt \qquad (3.4)$$

subject to the dynamic constraints given by (3.1) as well as to satisfy the system transient, steady state and stability specifications. Here, Q_1 is a positive definite diagonal matrix, γ is equal to 1. By adjusting the element values of Q_1 we can weigh the relative importance of each of the system states. Thus, by increasing q_{1i} we attach more significance to x_i, by making q_{1i} zero we indicate the state x_i is of no concern whatsoever. Here all the states have the same importance. Therefore Q has been choosen to be the identity matrix.

The transient requirement is met when the generation rate limit is kept within Ψ pu power/sec. The generation rate limit is not fixed and varies from unit to unit. The transient specification can be mathematically expressed in terms of the state variables as follows:

$$\frac{1}{T_t} x_2 - \frac{1}{T_t} x_3 \leq \Psi \qquad (3.5)$$

In fact the R.H.S. of (3.5) is the R.H.S. of the third equation in (3.1).

The steady state requirement is met when the frequency deviation is equal to zero at the end of the control period provided that the disturbed area can fully accommodate its load changes. This is achieved as a result of the introduction of the integral action in the control law.

The stability requirement is met whenever the control parameters result in an asymptotically stable system with adequate relative stability.

To cast the problem of the LFC of a single area steam power system into a problem of parameter optimization, the system dynamics given by (3.1) can be written as

$$\dot{X}_1 = AX_1 - B_1 k_p CX_1 - B_1 k_1 C_1 \int_{t_0}^{t_f} X_1 dt + \Gamma_1 \Delta L \qquad (3.6)$$

Equation (3.6) is the result of the direct substitution of (3.2) into (3.1). Now, define a new variable

$$x_4 = C_1 \int_{t_0}^{t_f} X_1 dt \tag{3.7}$$

then, define an augmented vector

$$X^T = (x_1, x_2, x_3, x_4)$$

hence, the augmented dynamic system can be written as

$$\dot{X} = A\ X + \Gamma \Delta L \tag{3.8}$$

where

$$A = \left[\begin{array}{c|c} A_1 k_p C & -B_1 k_1 \\ \hline C_3 & 0 \end{array}\right], \quad \Gamma^T = (-\frac{1}{M}, 0, 0, 0)$$

with 0_3 being 3x1 row vector whose elements are all zeros. The augmented cost functional can be written as

$$J = \frac{1}{2} \int_{t_0}^{t_f} X^T Q X\ dt \tag{3.9}$$

in which

$$Q = \left[\begin{array}{c:c:c:c} q_1+\gamma k_p^{\;2} & & & \gamma k_p k_I \\ \hdashline & q_2 & & \\ \hdashline & & q_3 & \\ \hdashline \gamma k_p k_I & & & q_4+\gamma K_I^2 \end{array}\right]$$

Now the problem is ready to be solved by employing the technique which has been developed in chapter 2.

3.2.2 The solution of the optimization problem of the LFC of single area steam power system

The problem of the LFC of a SASPS can be summarized as follows:

find the optimal control parameters k_I or k_p-plus-k_I which minimize the cost functional

$$J = \frac{1}{2} \int_{t_0}^{t_f} X^T Q(k_p, k_I) X dt \tag{3.10}$$

subject to the dynamic equality constraints

$$\dot{X} = AX + \Gamma \, \Delta L \tag{3.11}$$

and to the inequality constraint

$$\frac{1}{T_t} x_2 - \frac{1}{T_t} x_3 \leq \Psi \tag{3.12}$$

The inequality constraint (3.12) can be converted into an equality constraint [69] by introducing a new variable

$$\dot{x}_5 = \left(\frac{1}{T_t} x_2 - \frac{1}{T_t} x_3 - \Psi\right)^2 H_1\left(\frac{1}{T_t} x_2 - \frac{1}{T_t} x_3 - \Psi\right) \qquad (3.13)$$

with

$$x_5(t_0) = x_5(t_f) = 0$$

in which

$$H_1(\alpha) = \begin{cases} 0 & \text{if } \alpha < 0 \\ 1 & \text{if } \alpha > 0 \end{cases}$$

Since the conjugate-gradient technique is employed here to solve the parametric optimization problem, there is no way to ensure that the constraint will not be violated during the computational procedure of finding the optimal parameters. Consequently, the requirement that $x_5(t_f)$ is equal to zero cannot be met. Thus a penalty is placed on $x_5(t_f)$ for being larger than zero. Therefore a modified cost functional can be written as follows:

$$J = \frac{1}{2} \int_{t_0}^{t_f} X^T QX \, dt + w_5 x_5^2(t_f) \qquad (3.14)$$

To handle a criterion of the form (3.14), the system differential equations and the constraint are augmented by an additional equation by introducing a new variable

$$\dot{x}_6 = \frac{1}{2} X^T Q X \qquad (3.15)$$

with

$$x_6(t_0) = 0$$

By defining a Hamiltonian

$$H = \sum_{i=1}^{6} \lambda_i \dot{x}_i \qquad (3.16)$$

and by applying Pontryagin's minimum principle, one can get the system canonic equations in for either integral or proportional-plus-integral control action.

i - The optimal integral controller

In the case of an integral controller, the system dynamics can be written as follows

$$\dot{x}_1 = -\frac{G}{M} x_1 + \frac{1}{M} x_3 - \frac{\Delta L}{M}$$

$$\dot{x}_2 = -\frac{E}{T_g} x_1 - \frac{1}{T_g} x_2 - \frac{1}{T_g} k_I x_4$$

$$\dot{x}_3 = \frac{1}{T_t} x_2 - \frac{1}{T_t} x_3 \qquad (3.17)$$

$$\dot{x}_4 = x_1$$

$$\dot{x}_5 = \left(\frac{1}{T_t} x_2 - \frac{1}{T_t} x_3 - \psi\right)^2 H_1\left(\frac{1}{T_t} x_2 - \frac{1}{T_t} x_3 - \psi\right)$$

$$\dot{x}_6 = \frac{1}{2} \sum_{i=1}^{3} q_i x_i^2 + \frac{1}{2} \gamma (k_I x_4)^2$$

with

$$x_i(t_0) = 0\ ,\quad i = 1,\ 2,\ \ldots,\ 6$$

and the system costate equations can be written as

$$\begin{aligned}
\dot{\lambda}_1 &= -\,q_1x_1 + \frac{G}{M}\,\lambda_1 + \frac{E}{T_g}\,\lambda_2 - \lambda_4 \\
\dot{\lambda}_2 &= -\,q_2x_2 + \frac{1}{T_g}\,\lambda_2 - \frac{1}{T_t}\,\lambda_3 - \frac{2}{T_t}\left(\frac{1}{T_t}\,x_2 - \frac{1}{T_t}\,x_3\right) H_1\lambda_5 \\
\dot{\lambda}_3 &= -\,q_3x_3 + \frac{1}{T_t}\,\lambda_3 + \frac{2}{T_t}\left(\frac{1}{T_t}\,x_2 - \frac{1}{T_t}\,x_3\right) H_1\lambda_5 \qquad (3.18) \\
\dot{\lambda}_4 &= -\,\gamma\, x_4k_I^2 + \frac{1}{T_g}\,k_I\lambda_2 \\
\dot{\lambda}_5 &= 0 \\
\dot{\lambda}_6 &= 0
\end{aligned}$$

with

$$\lambda_i(t_f) = 0, \qquad i = 1,\ 2,\ 3,\ 4.$$

$$\lambda_5(t_f) = 2w_5x_5(t_f)$$

$$\lambda_6\cancel{(t}_f) = 1.$$

and the gradient vector is given by

$$H_{k_I} = -\gamma x_4^2 k_I + \frac{1}{T_g} x_4 \lambda_2 \qquad (3.19)$$

The system canonic equations have been programmed for the computer with system parameters given by [15]

$M = .04$, $G = .01$, $T_g = .5$, $T_t = .5$, and $E = .03$

$\Delta L = -.005$ pu

$\Psi = 0.05$

and the weighting coefficients have been chosen as follows

$q_i = 1, \quad i = 1, 2, 3$

$\gamma = 1$

The penalty factor, in the case that $x_5(t_f)$ is greater than zero, has been chosen to be

$$w_5 = 1000.$$

Using the gradient as determined in (3.19) over

$$[t_0, t_f] = [0, 20 \text{ sec}]$$

conjugate gradient-descent has been accomplished with the program described in chapter 2. The initial guess of the control parameter k_I and the step size α_0 have been taken to be equal to zero and 10^{-2} respectively. After 2 iterations (13 trials), the optimal integral gain parameter has been found to be as follows

$$k_I = .01703$$

with a corresponding cost

$$J = 0.02119$$

with

$$||H_{k_I}|| = 0.16\text{x}10^{-3}$$

The augmented dynamic system eigenvalues are given as follows:

$$-0.769, \quad -2.9476, \quad -0.2665 \pm j0.824$$

Therefore, the optimal integral control parameter results in an asymptotically stable system.

ii - The optimal proportional-plus-integral controller

The problem of finding the optimal proportional-plus-integral control gain parameters is considered. Thus, equations number 1 and 6 in (3.17) are replaced by

$$\dot{x}_2 = -\frac{E}{T_g}x_1 - \frac{1}{T_g}x_2 - \frac{1}{T_g}k_px_1 - \frac{1}{T_g}k_Ix_4$$

and

$$\dot{x}_6 = \frac{1}{2} \sum_{i=1}^{4} q_i x_i^2 + \frac{1}{2} \gamma (k_I x_4 + k_p x_1)^2$$

to obtain the system dynamic equations in the case of a proportional-plus-integral control action. Equations 1 and 4 in (3.18) are also replaced by

$$\dot{\lambda}_1 = - q_1 x_1 - \gamma k_p^2 x_1 - \gamma k_p k_I x_4 + \frac{G}{M} \lambda_1 + \frac{E}{T_g} \lambda_2 - \lambda_4$$

and

$$\dot{\lambda}_4 = - \gamma k_I^2 x_4 - \gamma k_p k_I x_1 + \frac{1}{T_g} k_I \lambda_2$$

to get the system costate equations.

In this case the gradient vector components are given by

$$H_{k_p} = - \gamma (k_p x_1^2 + k_I x_1 x_4) + \frac{1}{T_g} x_1 \lambda_2$$

and

$$H_{k_I} = - \gamma (k_I x_4^2 + k_p x_1 x_4) + \frac{1}{T_g} x_4 \lambda_2$$

Using the gradient as determined in (3.19) over the same time interval with the same system's parameter and load disturbance, the conjugate-gradient descent has been accomplished with the algorithm described in chapter 2. The initial guess of the control parameters k_p, k_I and the step size have been taken simply to be zero, zero, and 10^{-3}. After 8 iterations (23 trials), the optimal control parameters are as follows

$$k_p = 0.0859$$

$$k_I = 0.02177$$

and the corresponding cost

$$J = 0.696\text{x}10^{-2}$$

with

$$||H_k|| = 0.2926\text{x}10^{-4}$$

The eigenvalues of the augmented system are given as follows

$$-3.769, \quad -0.1484 \pm j1.7641, \quad -0.18368$$

Therefore, the optimal proportional-plus-integral controller results in an asymptotically stable system. However, in designing a control system, it is not only required that the system be stable but also it is necessary that the system has adequate relative stability. Here, the optimal system relative stability is unsatisfactory since it has a pair of dominant complex -conjugate closed loop poles near the $j\omega$ axis. However, the problem of designing an optimal output feedback controller with satisfactory relative stability can be achieved by modifying the system structure. This can be realized by addition of a zero to the open-loop transfer function which has the effect of pulling the root locus to the left. Physically, the addition of a zero in the feedforward transfer function means the addition of derivative control to the system. Therefore, the case of studying the addition of derivative to the proportional-plus-integral controller is considered next.

iii -The optimal proportional-plus-integral-plus-derivative controller

The control law in this case can be written as follows

$$u = - k_p x_1 - k_I \int_{t_0}^{t_f} x_1 dt - k_d \dot{x}_1$$

or

$$u = - (k_p - \frac{G}{M} k_d) x_1 - \frac{1}{M} k_d x_3 - k_I x_4 + \frac{1}{M} k_d \Delta L \qquad (3.20)$$

By applying the procedure adopted in section (i) and (ii), the system dynamics equations are the same as in eqn. (3.17) except $\dot{x}_2$ which is replaced by

$$\dot{x}_2 = - (\frac{E}{T_g} + \frac{1}{T_g} k_p - \frac{G}{MT_g} k_d) x_1 - \frac{1}{T_g} x_2 - \frac{1}{MT_g} x_3 - \frac{1}{T_g} k_1 x_4 + \frac{1}{MT_g} \Delta L k_d$$

Also $\dot{x}_6$ is replaced by

$$\dot{x}_6 = \sum_{i=1}^{3} q_i x_i^2 + [(k_p - \frac{G}{M} k_d) x_1 + \frac{1}{M} k_d x_3 - k_I x_4 + \frac{1}{M} k_d \Delta L]^2$$

The system costate equations are also the same as in eqn. (3.18) except $\dot{\lambda}_1$ is replaced by

$$\dot{\lambda}_1 = - q_1x_1 - \gamma[(k_p - \frac{G}{M}k_d)x_1 + \frac{1}{M}k_dx_3 + k_Ix_4 - \frac{1}{M}k_d\Delta L](k_p - \frac{G}{M}k_d)$$

$$+ \frac{G}{M}\lambda_1 + (\frac{E}{T_g} + \frac{1}{T_g}k_p - \frac{G}{MT_g}k_d)\ \lambda_2 - \lambda_4$$

Also $\dot{\lambda}_3$ is replaced by

$$\dot{\lambda}_3 = - q_3x_3 - \frac{r}{M}k_p[(k_p - \frac{G}{M}k_d)x_1 + \frac{1}{M}k_dx_3 + k_Ix_4 - \frac{1}{M}k_d\Delta L]$$

$$- \frac{1}{M}\lambda_1 + \frac{1}{MT_g}k_d\lambda_2 + \frac{1}{T_t}\lambda_3 + \frac{2}{T_t}(\frac{1}{T_t}x_2 - \frac{1}{T_t}x_3)H_1\lambda_5$$

Moreover $\dot{\lambda}_4$ is replaced by

$$\dot{\lambda}_4 = - r\,k_I[(k_p - \frac{G}{M}k_d)x_1 + \frac{1}{M}k_dx_3 + k_Ix_4 - \frac{1}{M}k_d\Delta L]$$

$$- \frac{1}{T_g}k_I\ \lambda_2$$

The gradient vector components are given by

$$H_{k_p} = - rx_1[(k_p - \frac{G}{M}k_d)x_1 \pm \frac{1}{M}k_dx_3 + k_Ix_4 - \frac{1}{M}k_d\Delta L]$$

$$+ \frac{1}{T_g}x_1\lambda_2$$

$$H_{k_I} = - rx_4[(k_p - \frac{G}{M}k_d)x_1 + \frac{1}{M}k_dx_3 + k_Ix_4 - \frac{1}{M}k_d\Delta L] + \frac{1}{T_g}x_4\lambda_2$$

$$H_{k_d} = -r[(k_p - \frac{G}{M} k_d)\, x_1 + \frac{1}{M} k_d x_3 + k_I x_4 - \frac{1}{M} k_d \Delta L](- \frac{G}{M} x_1 + \frac{1}{M} x_3 - \frac{\Delta L}{M})$$

$$+ (- \frac{G}{MT_g} x_1 + \frac{1}{MT_g} x_3 - \frac{\Delta L}{MT_g})\, \lambda_2 \qquad (3.21)$$

Using the gradient given by equations (3.21) over the time interval

$$[t_0, t_f] = [0, 20\text{sec}]$$

the conjugate gradient is found with the technique described in chapter 2. The initial guess of the control parameter vector and the step size have been simply chosen equal to 0.107, 0.077, 0.08276 and 10^{-2} respectively. After 30 iterations (62 trials), the optimal control parameters have been found as follows

$$k_p = 0.111$$

$$k_I = 0.0790$$

$$k_d = 0.0863$$

and the optimum cost is

$$J = 0.18355 \times 10^{-2}$$

with

$$||H_K|| = 0.0102$$

The system's eigenvalues corresponding to the optimal control vector are given as follows

$$-0.724 \pm j0.6226, \text{ and } -1.4 \pm j2.588$$

Therefore, by employing a proportional-plus-integral-plus-derivative controller, a system with adequate relative stability as well as less state deviations has been established.

Quite often in designing a controller for a higher order system the control parameters are adjusted so that there will exist a pair of dominant complex-conjugate closed-loop poles. The presence of such poles in a stable system reduces the effect of such nonlinearities as dead zone and backlash. Therefore, a proportional-plus-integral-plus-derivative controller will be designed next so that the system will have a preassigned dominant pair of complex-conjugate poles and the rest of the poles will be preassigned negative real.

iv - Optimal proportional-plus-integral-plus-derivative controller by pole-assignment

The characteristic equation of a single steam area power system can be written as follows

$$\sum_{i=1}^{5} a_i s^{4-i} = 0 \qquad (3.22)$$

where

$$a_1 = 1$$

$$a_2 = \frac{1}{T_g} + \frac{1}{T_t} + \frac{G}{M} \qquad (3.23)$$

$$a_3 = \frac{G}{MT_g} + \frac{G}{MT_t} + \frac{1}{T_tT_g} + \frac{1}{MT_tT_g} k_d$$

$$a_4 = \frac{G + E}{MT_gT_t} + \frac{1}{MT_gT_t} k_p$$

$$a_5 = \frac{1}{MT_gT_t} k_I$$

By substituting the system parameters in (3.23), one can write (3.24) as follows

$$s^4 + 4.25s^3 + (5 + 100k_d)s^2 + (4+100k_p)\, s + 100k_I = 0 \qquad (3.24)$$

The problem posed is to find the optimal control parameters k_p, k_I, and k_d required to position the four poles of the plant plus the integrator at

$$-\ .5 \pm j0,\ \ -\ 1.25,\ \text{and}\ -\ 2\ ,\ \phi > 0$$

By formulating the desired characteristic equation and comparing it to the closed-loop one, one can get

$$k_p = -\ 0.039375 + 3.25k_d$$

$$k_I = -\ 0.01875 + 2.5k_d \qquad (3.25)$$

To find the optimal pole assignment control parameters, one can adopt the same procedure as in section (i). This results in a system canonic equations similar to the one given in section (iii) except that $\dot{x}_2$ is replaced by

$$\dot{x}_2 = - [\frac{E}{T_g} + \frac{1}{T_g}(- 0.039375 + 3.25k_d) - \frac{G}{MT_g} k_d]x_1$$

$$- \frac{1}{T_g} x_2 - \frac{1}{MT_g} x_3 - \frac{1}{T_g} (- 0.01875 + 2.5k_d) x_4 + \frac{1}{MT_g} \Delta L k_d$$

Also $\dot{\lambda}_1$ is replaced by

$$\dot{\lambda}_1 = - q_1 x_1 - \gamma[(- 0.039375 + 3.25k_d - \frac{G}{M} k_d) x_1 + \frac{1}{M} k_d x_3$$

$$+ (- 0.01875 + 2.5 k_d)x_4 \ \frac{1}{M} k_d \ \Delta L][3.25 \ \frac{G}{M})k_d - 0.039375]$$

$$+ \frac{G}{M} \lambda_1 + [\frac{E}{T_g} + \frac{1}{T_g} (- 0.039375 + 3.25k_d) - \frac{G}{MT_g} k_d] \lambda_2 - \lambda_4$$

, $\dot{\lambda}_3$ is replaced by

$$\dot{\lambda}_3 = - q_3 x_3 - \frac{\gamma}{M} [(- 0.039375 + 3.25k_d - \frac{G}{M} k_d) x_1 + \frac{1}{M} k_d x_3$$

$$+ (- 0.01875 + 2.5k_d)x_4 - \frac{1}{M} k_d \ \Delta L][- 0.039375 + 3.25k_d]$$

$$- \frac{1}{M} \lambda_1 + \frac{1}{MT_g} k_d \lambda_2 + \frac{1}{T_t} \lambda_3 + \frac{2}{T_t} (\frac{1}{T_t} x_2 - \frac{1}{T_t} x_3) H_1 \lambda_5$$

Finally $\dot{\lambda}_4$ is replaced by

$$\dot{\lambda}_4^* = - q_4 x_4 - \gamma[(- 0.039375 + 3.25k_d - \frac{G}{M} k_d) x_1 + \frac{1}{M} k_d x_3$$

$$+ (- 0.01875 + 2.5k_d)x_4 - \frac{1}{M} k_d \Delta L][- 0.01875 + 2.5k_d]$$

$$- \frac{1}{T_g} (- 0.01875 + 2.5k_d)\lambda_2$$

The gradient with respect to k_d is given as follows

$$H_{k_d} = - \gamma[(-0.039375 + 3.25k_d - \frac{G}{M} k_d) x_1 + \frac{1}{M} k_d x_3$$

$$+ (-0.01875 + 2.5k_d) x_4 - \frac{1}{M} k_d \Delta L][(3.25 - \frac{G}{M}) x_1 + \frac{1}{M} x_3$$

$$+ 2.5 x_4 - \frac{1}{M} \Delta L] + [\frac{3.25}{T_g} x_1 + \frac{G}{MT_g} + \frac{2.5}{T_g} x_4 - \frac{\Delta L}{MT_g}] \lambda_2$$

(3.26)

By choosing the initial guess of the control parameter k_d and the initial step size α_0 equal to 1×10^{-2} and 10^{-5} respectively, the optimal control parameter is found to be as follows

$$k_d = 0.096$$

and the optimal cost

$$J = 0.147 \times 10^{-2}$$

and the gradient

$$||H_{k_d}|| = 0.67 \times 10^{-2}$$

In conclusion, the problem of designing an optimal or suboptimal controller for the problem of LFC of single area steam power system has been considered. In the case of employing integral and/or proportional control action, the optimal control parameters k_I and/or k_p have been found to be independent of the load changing. In the case of adopting a PID control strategy, the optimal control parameters have been found to be strongly dependent on the load changing. The system response for different controllers in the case of disturbing the area by - 0.005 p.u., is shown in fig. [2]. Fig. [3] shows the system response for the optimal PID controller and the suboptimal one. They emphasize the figure of merit of adopting modern control theory; rather than the classical one, on designing system controllers.

In spite of the fact that the optimal or the suboptimal PID controllers give better performance than the optimal integral and/or proportional controllers, it is recommended that the optimal PI controller be used for the following reasons:

- The optimal PI control parameters are independent of the load changing
- It is simpler to design the hardware of the PI controller rather than that of the PID one
- The PID controller might produce an undesirable system performance in the case that the area control signal experiences a noise

component in its measurement.

3.3 Load frequency control of single area hydro power system

3.3.1 The problem formulation

A typical linearized model of a hydro area power system; (SAHPS) is shown in fig. (4) [15]. The system dynamics can be put in the state space form as follows:

$$\dot{X}_1 = A_1X_1 + B_1u + \Gamma_1\Delta_1 \qquad (3.27)$$

$$Y = C_1X_1$$

where

$$X_1^T = (\Delta\omega, \Delta x_g, \Delta_g, \Delta p_g)$$

in which

Δ_g = the deviation in the gate position in pu power

and

$$C_1^T = (1, 0, 0, 0) \quad , B_1^T = (1, 0, 0, 0)$$

$$\Gamma^T = (-\frac{1}{M}, 0, 0, 0)$$

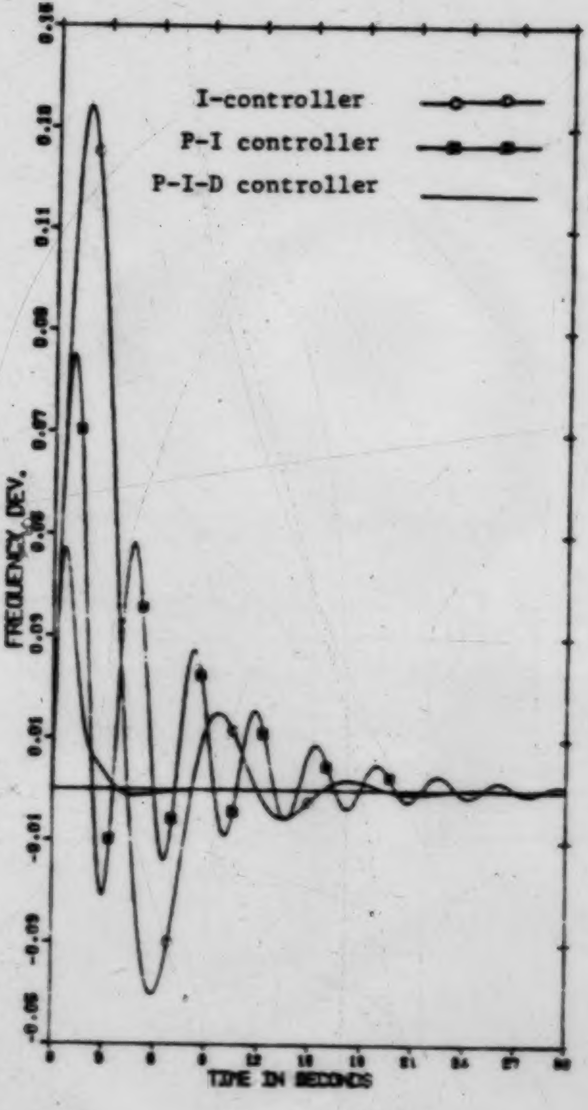

Fig. (2-a) Frequency deviation-time

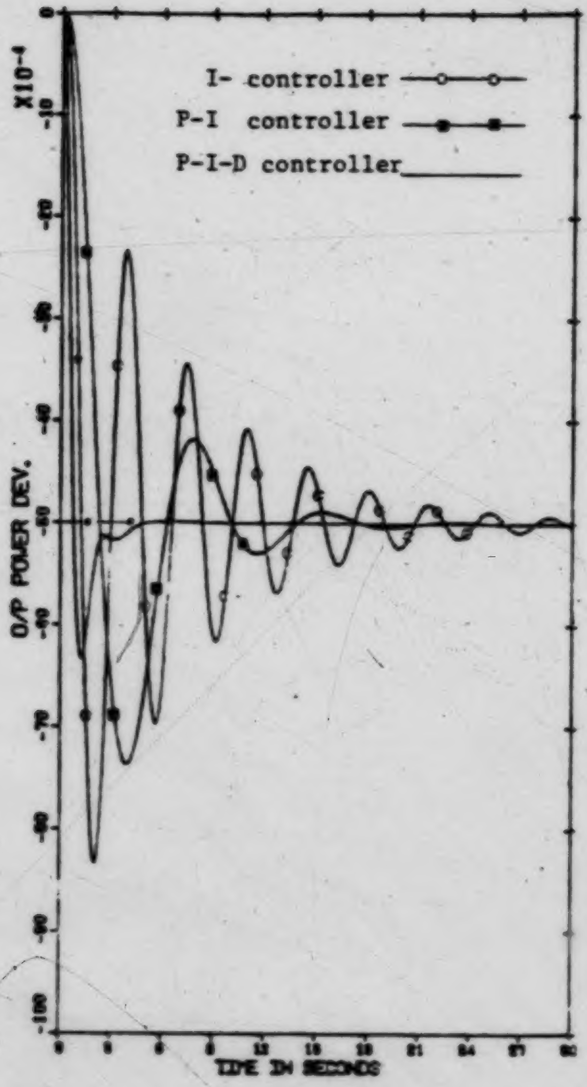

Fig. (2-b) Output power deviation-time

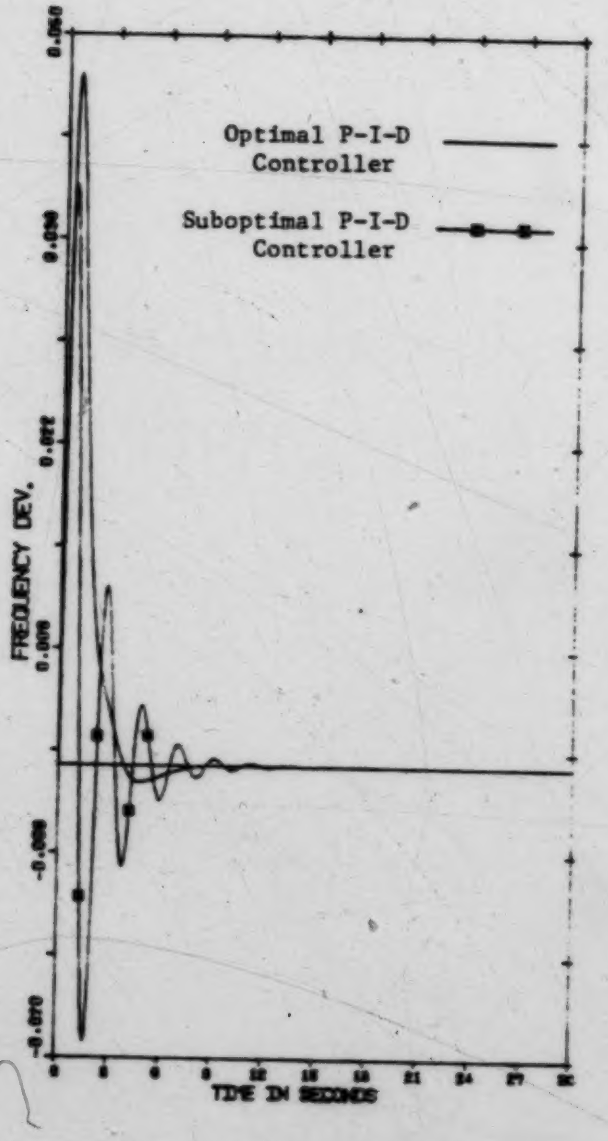

Fig. (3-a) Frequency deviation-time

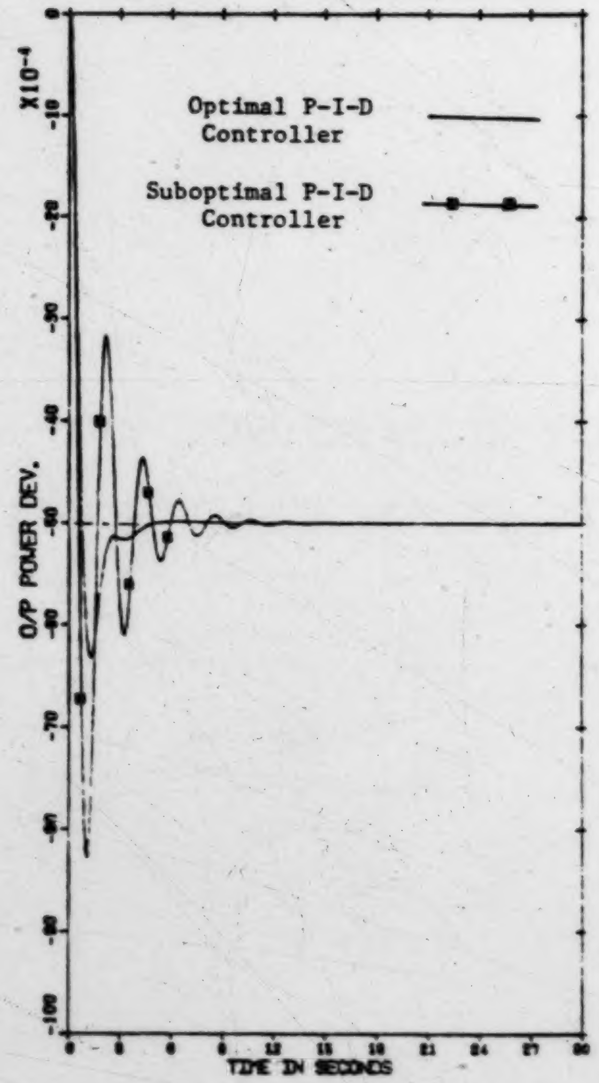

Fig. (3-b) Output power deviation-time

Comparison between the optimal and the suboptimal P-I-D controller on the dynamic response of single area power system

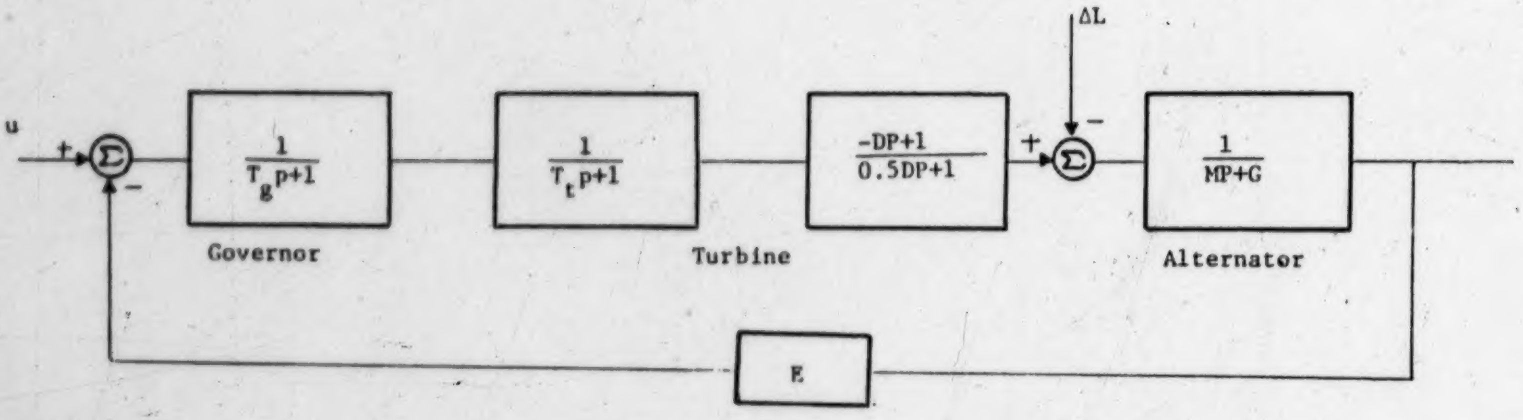

Figure (4). Block diagram of single hydro area power system

$$A_1 = \left[\begin{array}{c|c|c|c} -\frac{G}{M} & & & \frac{1}{M} \\ \hline -\frac{E}{T_g} & -\frac{1}{T_g} & & \\ \hline & \frac{1}{T_t} & -\frac{1}{T_t} & \\ \hline & -\frac{2}{T_t} & \frac{2}{D} + \frac{2}{T_t} & -\frac{2}{D} \end{array}\right]$$

The control law is specified to be of integral or proportional-plus-integral action. Therefore, the control law can be written in general, in terms of the system states as follows:

$$u = - k_p x_1 - k_I \int_{t_0}^{t_f} x_1 dt \tag{3.28}$$

To cast the problem in the normal state space form, a new variable x_5 is defined as follows:

$$\dot{x}_5 = x_1$$

then, define an augmented vector X by

$$X^T = (x_1, x_2, x_3, x_4, x_5) \tag{3.29}$$

hence, the augmented dynamic system can be written as follows:

$$\dot{X} = AX + \Gamma \Delta L \tag{3.30}$$

where

$$A = \left[\begin{array}{c:c:c:c:c} -\frac{G}{M} & & & \frac{1}{M} & \\ \hdashline -\frac{E}{T_g} - \frac{1}{T_g} k_p & -\frac{1}{T_g} & & & -\frac{1}{T_g} k_I \\ \hdashline & \frac{1}{T_t} & -\frac{1}{T_t} & & \\ \hdashline & -\frac{2}{T_t} & \frac{2}{D} + \frac{2}{T_t} & -\frac{2}{D} & \\ \hdashline 1 & & & & \end{array}\right]$$

and

$$\Gamma^T = (-\frac{1}{M}, 0, 0, 0, 0)$$

The problem posed is to find the optimal values of k_I and $(k_p k_I)^T$ which minimize the cost functional

$$J = \frac{1}{2} \int_{t_0}^{t_f} (X^T Q X + r u^2) dt \qquad (3.31)$$

such that the dynamic system given by (3.30) is asymptotically stable.

3.3.2 The optimal solution

To handle a criterion of the form (3.31), the system differential equations are augmented by an additional equation. This equation is defined by

$$\dot{x}_6 = \frac{1}{2} X^T Q X + \frac{1}{2} r u^2 \tag{3.32}$$

with

$$x_6(t_0) = 0$$

By defining a Hamiltonian

$$H = \sum_{i=1}^{6} \lambda_i \dot{x}_i \tag{3.33}$$

then, by applying the Pontryagin minimum principle, one can get the system canonic equations. By solving these equations as described in chapter 2, one can get the optimal control parameters.

i - The optimal integral controller

In the case of employing an integral controller, the control law which represents the speed changer commands can be written as follows:

$$u = - k_I \int_{t_0}^{t_f} x_1 dt \tag{3.34}$$

Therefore, following a step load change, a new steady state is attained after the speed changer command; u, has reached constant value. But this evidently requires that the integrand in (3.34) be zero, i.e.,

$$x_1^{ss} = 0 \tag{3.35}$$

which is the basic proposition of the LFC in single area systems. The application of the Pontryagin minimum principle results in the following system canonic equations

$$\dot{x}_1 = -\frac{G}{M}x_1 + \frac{1}{M}x_4 - \frac{\Delta L}{M}$$

$$\dot{x}_2 = -\frac{E}{T_g}x_1 - \frac{1}{T_g}x_2 - \frac{1}{T_g}k_I x_5$$

$$\dot{x}_3 = \frac{1}{T_t}x_2 - \frac{1}{T_t}x_3$$

$$\dot{x}_4 = -\frac{2}{T_t}x_2 + \left(\frac{2}{D} + \frac{2}{T_t}\right)x_3 - \frac{2}{D}x_4 \tag{3.36}$$

$$\dot{x}_5 = x_1$$

$$\dot{x}_6 = \frac{1}{2}\sum_{i=1}^{5} q_i x_i^2 + \frac{1}{2}r(k_I x_5)^2$$

with

$$x_i(t_0) = 0 \quad i = 1, 2, \ldots, 6$$

and

$$\dot{\lambda}_1 = -q_1x_1 + \frac{G}{M}\lambda_1 + \frac{E}{T_g}\lambda_2 - \lambda_5$$

$$\dot{\lambda}_2 = -q_2x_2 + \frac{1}{T_g}\lambda_2 - \frac{1}{T_t}\lambda_3 + \frac{2}{T_t}\lambda_4$$

$$\dot{\lambda}_3 = -q_3x_3 + \frac{1}{T_t}\lambda_3 - \left(\frac{2}{D} + \frac{2}{T_t}\right)\lambda_4$$

$$\dot{\lambda}_4 = - q_4 x_4 - \frac{1}{M} \lambda_1 + \frac{2}{D} \lambda_4 \qquad (3.37)$$

$$\dot{\lambda}_5 = - r\, k_I^2 x_5 + \frac{1}{T_g} k_I \lambda_2$$

$$\dot{\lambda}_6 = 0.$$

with

$$\lambda_1(t_f) = 0 \qquad i = 1, 2, \ldots, 5$$

$$\lambda_6(t_f) = 1$$

with a gradient

$$H_{k_I} = - r x_5^2 k_I + \frac{1}{T_g} x_5 \lambda_2 \qquad (3.38)$$

The system canonic equations have been programmed to be solved with system parameters given by [15]

$M = 0.03$, $G = 0.003$, $T_g = 1{,}2$, $T_t = 0.5$, $D = 0.5$, and $E = 0.013$

and a step load disturbance given as follows

$\Delta L = + 0.005$pu

and the weighting coefficients have been chosen as follows

$q_i = 1, 2, \ldots, 4$

$r = 1$

Using the gradient as in (3.38) over

$[t_0, t_f] = [0, 20\text{sec}]$

with an initial guess for the control parameter and the step size equal to zero and 10^{-4} respectively, the conjugate-gradient-descent has been accomplished in 2 iterations (9 trials). The optimal integral gain parameter has been found to be as follows

$k_I = 0{,}00355$

and the corresponding cost is

$J = 0.19211$

with a gradient norm

$||H_{k_I}|| = 0.242 \times 10^{-3}$

The augmented dynamic system eigenvalues are given as follows

$-0.1586 \pm j0.4945$ and $-3.253 \pm j0.1444$

Therefore, the optimal integral controller results in an asymptotically stable system.

ii - The optimal proportional-plus-integral-controller

In the case of adopting a proportional-plus-integral control action, the control law can be written as follows

$$u = -k_p x_1 - k_I \int_{t_0}^{t_f} x_1 dt \tag{3.39}$$

Following a step load change, at steady state, the speed changer command is constant. This requires that the integrand, the frequency deviation, in (3.39) be zero.

The augmented system dynamic equations is the same as in section (i); eqn. (3.36), except that $\dot{x}_2$ is replaced by

$$\dot{x}_2 = -\left(\frac{E}{T_g} + \frac{1}{T_g} k_p\right) x_1 - \frac{1}{T_g} x_2 - \frac{1}{T_g} k_I x_5$$

and $\dot{x}_6$ is replaced by

$$\dot{x}_6 = \frac{1}{2} \sum_{i=1}^{5} q_i x_i^2 + \frac{1}{2} r (k_p x_1 + k_I x_5)^2$$

with the same initial conditions.

Now, by defining a Hamiltonian as follows

$$H = \sum_{i=1}^{6} \lambda_i \dot{x}_i$$

then, by applying the Pontryagin minimum principle, one can get the costate equations as in section (i); eqn. (3.37), except that $\dot{\lambda}_1$ is replaced by

$$\dot{\lambda}_1 = - q_1 x_1 - r k_p (k_p x_1 + k_I x_5) + \frac{G}{M} \lambda_1 + (\frac{E}{T_g} + \frac{1}{T_g} k_p) \lambda_2 - \lambda_5$$

and the gradient vector components are as follows

$$H_{k_p} = - r x_1 (k_p x_1 + k_I x_5) + \frac{1}{T_g} x_1 \lambda_2$$

and (3.40)

$$H_{k_i} = - r x_5 (k_p x_1 + k_I x_5) + \frac{1}{T_g} x_5 \lambda_2$$

Using the gradient given by equations (3.40) over the time interval

$$[t_0, t_f] = [0, 20\text{sec}]$$

The control parameters were found in 9 iterations (30 trials). The initial guess of the control parameter vector and the step size were taken equal to zero and . 1 x 10^{-5} respectively. The optimal control parameters are given as follows

$$k_p = 0.0124$$

$$k_I = 0.00347$$

and the corresponding cost is given as follows

$J = 0.1355$

with a gradient norm

$||H_k|| = 0.2976 \times 10^{-4}$

The augmented dynamic system eigenvalues are given as follows

$- 0.0853 \pm j0.714$ and $- 3.405 \pm j0.843$

Therefore, the optimal proportional-plus-integral controller results in an asymptotically stable system. The optimal proportional-plus-integral controller (OPIC) is better than the optimal integral controller (OIC) in the sense that the system overall cost is smaller. On the other hand, the system relative stability, in the case of employing the OIC, is better than the system relative stability in the case of employing the OPIC. Both the OI and OPI control parameters have been found to be independent of the load changing. Fig. (5) shows the system response in the case of a -0,005 p.u. load change.

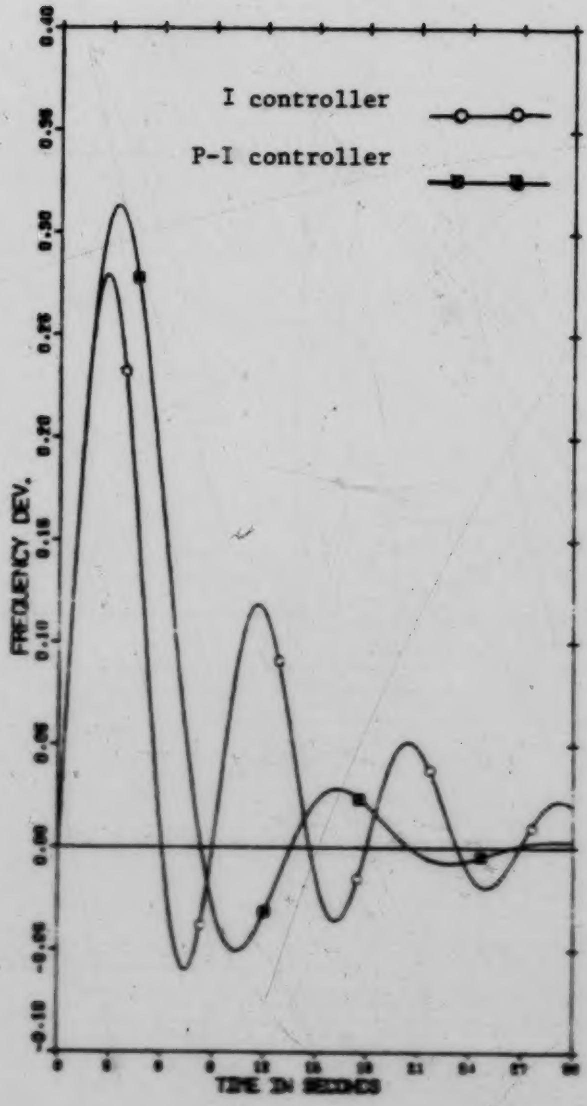

Fig. (5-a) Frequency deviation-time

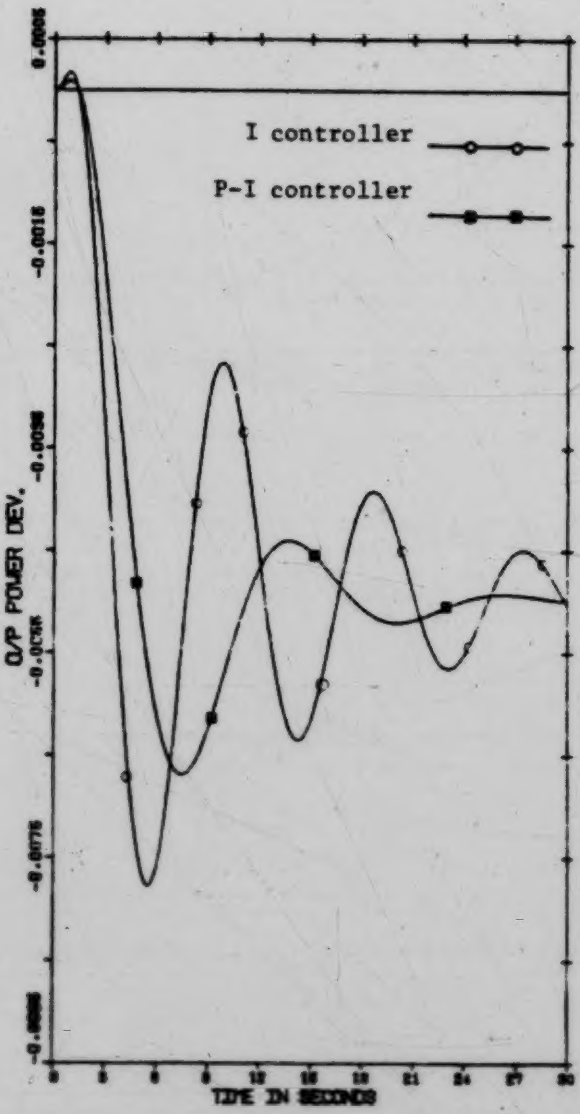

Fig. (5-b) Output power deviation-time

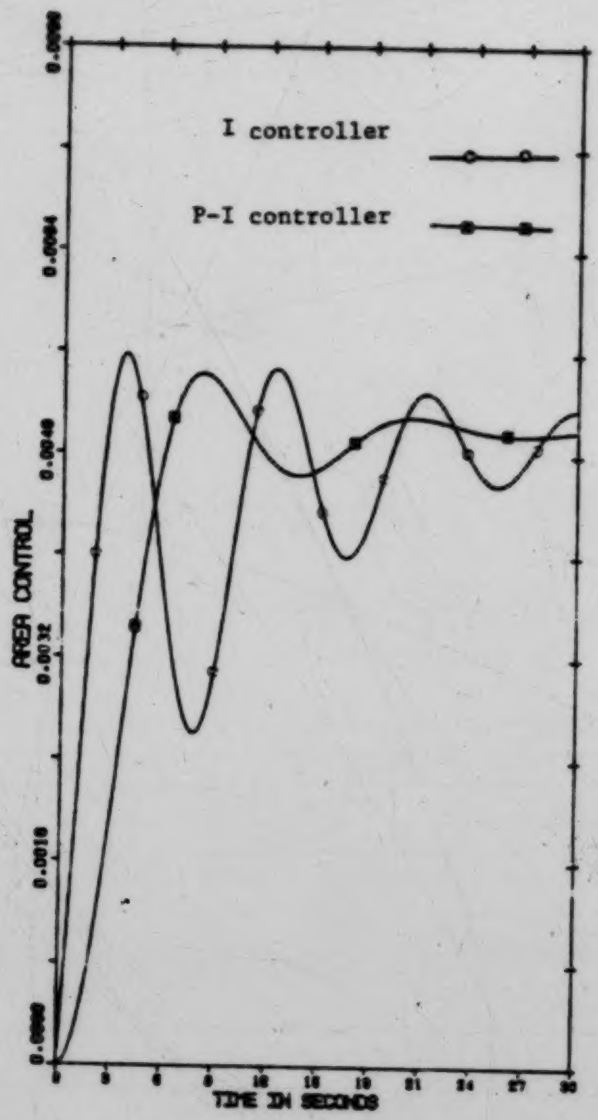

Fig. (5-c) Area control-time

Dynamic response of a single hydro area power system

Chapter 4

Load Frequency Control of Multi Area Interconnected Power System

4.1 Introduction

A multi area interconnected power system is one in which each area of the system is expected to adjust its own generation to absorb its own load changes. There is no attempt to have a complete coordination between all the areas of the interconnected system. Each area deals primarily with its adjacent neighbors. Tie-line power flow between adjacent areas are scheduled and maintained.

The load frequency control of an area of an interconnected power system relies on an operating schedule which is often prepared 24 hours ahead of time. This schedule indicates the expected demand profile of the area as well as the area commitment to its adjacent areas. The area commitment is the tie-line power interchange which should be maintained at a certain point of time. These values are fed to the area controller as governor set points.

In the normal mode of operation, the area prevailing frequency and tie-line power are compared to the area set point, scheduled frequency and tie-line power, every 2-5 seconds. An area control error (ACE) is computed according to the following equation

$$ACE = \Delta P_t + \beta \Delta f$$

where

ΔP_t = area net interchange tie-line power deviation

Δf = frequency deviation

β = area bias-setting

The ACE is converted to a train of pulses of raise/lower commands which drives the speed changer of the governing system in accordance with the sign of the ACE. A negative ACE causes generation to increase and vice versa. The speed governor shaft position will change in steps. At the end of the control period the speed governing characteristic will be shifted to a new position, a new set point, which results in matching the load demand with the load consumption.

Therefore, the governor set points are continuously altered to keep area frequency and tie-line power at their scheduled values. The transition from one set point to another represents a step change in the load demand which itself introduces a frequency transient and tie-line power disturbances before settling down to their scheduled values. Therefore, it is necessary to design the area's controllers with control gain parameters such that the area's transients are minimized as well as the area's steady state specifications being met.

The problem of designing area frequency regulators using conventional and modern control theory has been the subject of numerous studies since the earliest days of generation [5-30]. A significant group of results covering various aspects of this problem are available in the literature, (some of the relevant references are given at the end of this dissertation). The conventional LFC approach often employs the tie-line bias concept in designing the area's controller which has proportional and/or integral action. This type of control is extensively used in practice, inspite of all techniques which have been developed, recently, employing modern control theory [5]. The reason for that is the fact that most of these recent techniques have developed linear feedback controls which are functions of all the system states as well as system disturbances [6], [7]. Therefore, it was necessary to design an observer to realize these kinds of controls [8]. Once an observer is introduced into the system, the cost will be increased and the control is no longer optimal [9]. Another important reason is that a control which depends upon all the system states needs that some of these states to be telemetered since the areas of interconnected power systems are usually spread over large geographical territories. This is why in practice, control engineers prefer to use the conventional control to the advanced one inspite of the fact that the latter improves system transient performance.

Calovic [10], [11] was the first to narrow the gap between the conventional and the advanced LFC by adopting Porter's idea [12] of introducing the integral of the area output states as a part of the control design. However, the proportional part of the control was a function of all system states and an observer was needed to realize such a control. Here, modern control theory is implemented to find the optimal control parameters, of the integral and the proportional parts of the control law in a systematic way in order to meet the systems transient and steady specifications, as well as to conform to what control engineers practice in the real world of load-generation control.

4.2 Decentralized optimum load frequency control of interconnected power systems

The case considered in this dissertation is that of two mixed area interconnected power systems (TMAIPS) which may be regarded as a representation of a particular area which undergoes a disturbances, connected, via a tie-line, to the rest of the system. The disturbed area is assumed to be subjected to a step load change ΔL. The justification of assuming that interconnected power systems are subjected to deterministic load disturbances, inspite of the fact that they are normally subjected to random disturbances, is that modern LFC systems are designed with filters [13] in order to remove the purely random portion of the regulating responsibility which generation can not follow, leaving the deterministic components for the system units to follow.

Once an area has experienced a load change; e.g. load increase, a chain of events ensues as follows:

(i) The disturbed area speed begins to decrease as the increased load demand is supplied from the stored kinetic energy

(ii) The tie-line phase angle increases and unscheduled tie-line power flows to the disturbed area

(iii) The remote area speed begins to decrease, as the disturbed area unscheduled tie-line power represents a remote area load increase

(iv) The governors on both areas sense the change in the speed and each area responds in proportion to its natural governing characteristic

(v) The supplementary regulators come into action either after the governors have ceased operating or during their operation.

The approach which will be adopted in designing the supplementary regulators depends on whether they come into action before or after the governors have stopped operating. Load frequency control operation in which the governors cease operating before the action of the supplementary regulators will be referred to as non-interactive governor-supplementary regulator load frequency control. The case in which the supplementary regulators come into action during the operation of the governors will referred to be as interactive governor-supplementary regulator load frequency control.

Mathematical Model

A linearized model of two mixed area interconnected power system (TMAIPS) may be written as [8]:

$$\dot{X}^1 = A_1 X^1 + B_1 U + \Gamma_1 \Delta L$$

$$Y = C_1 X^1 \qquad (4.1)$$

in which

$$A_1 = \left[\begin{array}{c:c:c:c:c:c:c:c}
-\frac{G_1}{M_1} & & -\frac{1}{M_1} & & & \frac{1}{M_1} & & \\ \hdashline
 & -\frac{G_2}{M_2} & \frac{1}{M_1} & & & & & \frac{1}{M_2} \\ \hdashline
T_{12} & -T_{12} & & & & & & \\ \hdashline
-\frac{E_1}{T_{g1}} & & & -\frac{1}{T_{g1}} & & & & \\ \hdashline
 & -\frac{E_2}{T_{g2}} & & & -\frac{1}{T_{g2}} & & & \\ \hdashline
 & & & \frac{1}{T_{t1}} & & -\frac{1}{T_{t1}} & & \\ \hdashline
 & & & & \frac{1}{T_{t2}} & & -\frac{1}{T_{t2}} & \\ \hdashline
 & & & & -\frac{2}{D_2} & & \frac{2}{T_{t2}} + \frac{2}{D_2} & -\frac{2}{D_2}
\end{array}\right]$$

$$B_1^T = \left[\begin{array}{c:c:c:c:c:c:c:c} 0 & 0 & 0 & -\frac{1}{T_{g1}} & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & -\frac{1}{T_{g2}} & 0 & 0 & 0 \end{array}\right]$$

$$\Gamma_1^T = \left[\begin{array}{c:c:c:c:c:c:c:c} -\frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$C_1 = \left[\begin{array}{c:c:c:c:c:c:c:c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

where

x_1^1 = the frequency deviation of area 1

x_2^1 = the frequency deviation of area 2

x_3^1 = the power angle deviation

x_4^1 = the deviation in the governor position of area 1

x_5^1 = the deviation in the governor position of area 2

x_6^1 = the deviation in mechanical output of area 1

x_7^1 = the deviation in the gate position of area 2

x_8^1 = the deviation in mechanical output of area 2

4.2.1 The optimal non-interactive governor-supplementary regulator

In this mode of LFC operation the interconnected system settles down at a new frequency common to the areas of the interconnected system and a new tie-line flow before the supplementary regulator comes into action the case of a load increase; e.g. in area 1, ΔL, the system common frequency is given [14]

$$f = f^s + \frac{-\Delta L}{2\pi[(G_1+E_1)+(G_2+E_2)]} \qquad (4.2a)$$

and the tie-line power

$$P_{t_{12}} = P^s_{t_{12}} + \frac{-\Delta L(G_2 + E_2)}{(G_1+E_1)+(G_2+E_2)} \qquad (4.2b)$$

in which

f^s = the scheduled frequency

$P^s_{t_{12}}$ = the scheduled tie-line power

It is desired to design an optimal supplementary regulator with proportional-plus-integral control of the area control error such that system frequency and tie-line power are maintained at their scheduled values. In terms of the system's state variables the control law can be written as:

$$u(t) = - K_p \, ace(t) - K_I \int_{t_0}^{t_f} act(t)dt \qquad (4.3)$$

in which

$$ace(t) = D_1 x^1(t)$$

where

$$K_P = \text{diag}\ (k_{p1}, k_{p2})$$

$$K_I = \text{diag}\ (k_{I1}, k_{I2})$$

$$D_1 = \begin{bmatrix} \beta_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The control parameters K_P and K_I can be expressed mathematically as:

$$\begin{aligned} \dot{k}_{pi} &= 0 \qquad i = 1,2 \\ \dot{k}_{Ii} &= 0 \qquad i = 1,2 \end{aligned} \tag{4.4}$$

The problem posed is to find the optimal control parameters k_{Pi} and k_{Ii}, $i = 1,2$, which minimize the system's transient and the control action such that the system's steady state, dynamic limit and area's decentralization are met.

Minimization of the system's transient and the control effort can be accomplished by minimizing the cost functional

$$J_1 = \frac{1}{2} \int_{t_0}^{t_f} (X^{1^T} Q_1 X^1 + U^T R U)dt \tag{4.5}$$

with respect to the control parameters K_P and K_I subject to the dynamic constraints given by:

$$\dot{X}^1 = A_1 X^1 + B_1 U + \Gamma_1 \Delta L \qquad (4.6)$$

in which A_1, B_1, Γ_1 are defined in (4.1).

Here Q_1 and R are diagonal positive definite matrices of dimension 8 and 2 respectively.

The dynamic requirement is met when the steam area generation limit is kept within ψ p.u. mw/sec. the generation rate limit is not fixed and varies from unit to unit ($60\ \psi = 2\text{-}5$). It can be expressed mathematically in terms of the system's state variables as:

$$F_1 X^1 \leq \psi \qquad (4.7)$$

in which

$$F_1 = \left(0 \quad 0 \quad 0 \quad \frac{1}{T_{t1}} \quad 0 \quad -\frac{1}{T_{t1}} \quad 0 \quad 0 \right)$$

Ideally, to ensure the nonintervention principle of the interconnected load frequency control operation, that is, power generation must be altered only in the disturbed area; area frequency bias setting equal to the area natural governing characteristic must be employed in the area control error equation. When this condition is fulfilled, the control error, in the case of employing non-interactive governor-supplementary regulator, of the remote area will equal zero and its supplementary regulator remains inoperative.

To find the optimal control parameters K_I and K_P, the system's dynamics given by equation (4.6) can be written as:

$$\dot{X}^1 = A_1X^1 - B_1K_PDX^1 - B_1K_1D \int_{t_0}^{t_f} x^1dt + \Gamma_1\Delta L \tag{4.8}$$

Equation (4.8) is the result of the direct substitution of equation (4.3) into equation (4.8). Now define a new variable z by

$$Z = D_1 \int_{t_0}^{t_f} X^1dt \tag{4.9}$$

then, define an augmented vector X such that

$$\begin{aligned} x_i &= x_i^1 \qquad i = 1,..,8 \\ x_{8+j} &= z_j \qquad j = 1,2 \end{aligned} \tag{4.10}$$

hence, the augmented dynamic system and the cost functional can be written as

$$\dot{X} = AX + \Gamma\ \Delta L \tag{4.11}$$

$$J = \frac{1}{2} \int_{t_0}^{t_f} X^TQX^Tdt \tag{4.12}$$

where

$$A = \left[\begin{array}{c|c} A_1-B_1K_PD_1 & -B_1K_I \\ \hline D_1 & 0 \end{array}\right]$$

with 0 being the 2x2 matrix whose elements are all zeros,

$$\Gamma^T = (\Gamma_1^T, 0, 0)$$

and

$$Q = \left[\begin{array}{c:c} Q_1 + D^T K_P^T R K_P D_1 & D_1^T K_P^T R K_I \\ \hdashline K_I^T R K_P D & Q_2 + K_I^T R K_I \end{array}\right]$$

in which Q_2 is a diagonal positive definite matrix with dimension 2.

The inequality constraints given by equation (4.7) can be written as

$$FX \leq \psi \tag{4.13}$$

in which

$$F^T = (F_1^T \quad 0 \quad 0)$$

The optimal solution

The inequality constraints given by equation (4.13) can be converted into equality constraints, as in Chapter 3, by defining a new variable x_{11} such that

$$\dot{x}_{11} = (FX)^2 \, H(FX), \; H(FX) = \begin{cases} 0 & FX \leq 0 \\ 1 & FX > 0 \end{cases} \tag{4.14}$$

To handle a criterion of the form (4.12), the system differential equations and the constraint are augmented by an additional equation as follows

$$\dot{x}_{12} = \frac{1}{2}\ X^T Q X \tag{4.15}$$

Define the Hamiltonian

$$H = \sum_{i=1}^{12} \lambda_i \dot{x}_i \tag{4.16}$$

then, by applying Pontryagin's minimum principle, one can get the system canonic equations. In the case of employing proportional-plus-integral control action, the system dynamics can be written as follows

$$\dot{x}_1 = -\frac{G_1}{M_1} x_1 - \frac{1}{M_1} x_3 + \frac{1}{M_1} x_6 - \frac{\Delta L_1}{M_1}$$

$$\dot{x}_2 = -\frac{G_2}{M_2} x_2 + \frac{1}{M_2} x_3 + \frac{1}{M_2} x_8 - \frac{\Delta L_2}{M_2}$$

$$\dot{x}_3 = T_{12}\ x_1 - T_{12}\ x_2$$

$$\dot{x}_4 = -\frac{E_1}{T_{g1}} x_1 - \frac{1}{T_{g1}} x_4 - \frac{\beta_1 k_{P1}}{T_{g1}} x_1 - \frac{k_{P1}}{T_{g1}} x_3\ \frac{k_{I1}}{T_{g1}} x_9$$

$$\dot{x}_5 = -\frac{E_2}{T_{g2}} x_2 - \frac{1}{T_{g2}} x_5 - \frac{\beta_2 k_{P2}}{T_{g2}} x_2 + \frac{k_{P2}}{T_{g2}} x_3 - \frac{k_{I2}}{T_{g2}} x_{10}$$

$$\dot{x}_6 = \frac{1}{T_{t1}} x_4 - \frac{1}{T_{t1}} x_6$$

$$\dot{x}_7 = \frac{1}{T_{t2}} x_5 - \frac{1}{T_{t2}} x_7$$

$$\dot{x}_8 = -\frac{2}{T_{t2}} x_5 + \frac{2}{(T_{t2}+D_2)} x_7 - \frac{2}{D_2} x_8$$

$$\dot{x}_9 = \beta_1 x_1 + x_3$$

$$\dot{x}_{10} = \beta_2 x_2 - x_3$$

$$\dot{x}_{11} = \left(\frac{1}{T_{t1}} x_4 - \frac{1}{T_{t1}} x_6 - \psi\right)^2 H_1\left(\frac{1}{T_{t1}} x_4 - \frac{1}{T_{t1}} x_6 - \psi\right)$$

$$\dot{x}_{12} = \frac{1}{2} \sum_{i=1}^{10} q_i x_i^2 + \frac{1}{2} r_1 [k_{P1}(\beta_1 x_1 + x_3) + k_{I1} x_9]^2$$

$$+ \frac{1}{2} r_2 [k_{P2}(\beta_2 x_2 - x_3) + k_{I2} x_{10}]^2 \qquad (4.16)$$

with

$$x_1(t_0) = x_2(t_0) = - \frac{\Delta L}{(E_1+G_1)+(E_2+G_2)}$$

$$x_3(t_0) = - \frac{\Delta L(E_2+G_2)}{(E_1+G_1)+(E_2+G_2)}$$

$$x_4(t_0) = x_6(t_0) = - E_1 x_1(t_0)$$

$$x_5(t_0) = x_7(t_0) = x_8(t_0) = -E_2 x_2(t_0)$$

$$x_i(t_0) = 0 \qquad i = 9,10,..,12$$

and the system costate equation can be written as

$$\dot{\lambda}_1 = -q_1 x_1 - r_1 \beta_1 k_{P1}(\beta_1 k_{P1} x_1 + k_{P1} x_3 + k_{I1} x_9)$$

$$+ \frac{G_1}{M_1} \lambda_1 - T_{12} \lambda_3 + \left(\frac{E_1+\beta_1 k_{P1}}{T_{g1}}\right) \lambda_4 - \beta_1 \lambda_9$$

$$\dot{\lambda}_2 = - q_2 x_2 - r_2 \beta_2 k_{P2} (\beta_2 k_{P2} x_2 - k_{P2} x_3 + k_{I2} x_{10})$$

$$+ \frac{G_2}{M_2} \lambda_2 + T_{12} \lambda_3 + (\frac{E_2 + \beta_2 k_{P2}}{T_{g2}}) \lambda_5 - \beta_2 \lambda_{10}$$

$$\dot{\lambda}_3 = - q_3 x_3 - r_1 k_{P1} (\beta_1 k_{P1} x_1 + k_{P1} x_3 + k_{I1} x_9)$$

$$+ r_2 k_{P2} (\beta_2 k_{P2} x_2 - k_{P2} x_3 + k_{I2} x_{10}) + \frac{1}{M_1} \lambda_1$$

$$- \frac{1}{M_2} \lambda_2 + \frac{k_{P1}}{T_{g1}} \lambda_4 - \frac{k_{P2}}{T_{g2}} \lambda_5 - \lambda_9 + \lambda_{10}$$

$$\dot{\lambda}_4 = - q_4 x_4 + \frac{1}{T_{g1}} \lambda_4 - \frac{1}{T_{t1}} \lambda_6 - \frac{2}{T_{t1}} (\frac{1}{T_{t1}} x_4 - \frac{1}{T_{t1}} x_6 - \psi) H_1 \lambda_{11}$$

$$\dot{\lambda}_5 = - q_5 x_5 + \frac{1}{T_{g2}} \lambda_5 - \frac{1}{T_{t2}} \lambda_7 + \frac{2}{T_{t2}} \lambda_8$$

$$\dot{\lambda}_6 = - q_6 x_6 - \frac{1}{M_1} \lambda_1 + \frac{1}{T_{t1}} \lambda_6 + \frac{2}{T_{t1}} (\frac{1}{T_{t1}} x_4 - \frac{1}{T_{t1}} x_6 - \psi) H_1 \lambda_{11}$$

$$\dot{\lambda}_7 = - q_7 x_7 + \frac{1}{T_{t2}} \lambda_7 - (\frac{2}{T_{t2}} + \frac{2}{D_2}) \lambda_8$$

$$\dot{\lambda}_8 = - q_8 x_8 - \frac{1}{M_2} \lambda_2 + \frac{2}{D_2} \lambda_8$$

$$\dot{\lambda}_9 = - q_9 x_9 - r_1 k_{I1} (\beta_1 k_{P1} x_1 + k_{P1} x_3 + k_{I1} x_9) + \frac{k_{I1}}{T_{g1}} \lambda_4$$

$$\dot{\lambda}_{10} = - q_{10} x_{10} - r_2 k_{I2} (\beta_2 k_{P2} x_2 - k_{P2} x_3 + k_{I2} x_{10}) + \frac{k_{I2}}{T_{g2}} \lambda_5$$

$$\dot{\lambda}_{11} = 0$$

$$\dot{\lambda}_{12} = 0 \qquad (4.17)$$

with

$$\lambda_i(t_f) = 0 \qquad i=1,2,\ldots,10$$

$$\lambda_{11}(t_f) = 2w_{11}x_{11}(t_f)$$

$$\lambda_{12}(t_f) = 1$$

The components of the gradient vector are given by:

$$H_{k_{P1}} = -r_1(\beta_1 x_1+x_3)(k_{P1}\beta_1x_1+k_{P1}x_3+k_{I1}x_9) + \frac{1}{T_{g1}}(\beta_1x_1+x_3)\lambda_4$$

$$H_{k_{P2}} = -r_2(\beta_2x_2-x_3)(k_{P2}\beta_2x_2-k_{P2}x_3+k_{I2}x_{10}) + \frac{1}{T_{g2}}(\beta_2x_2-x_3)\lambda_5$$

$$H_{k_{I1}} = -r_1x_9(k_{P1}\beta_1x_1+k_{P2}x_3+k_{I1}x_9) + \frac{1}{T_{g1}}x_9\lambda_4$$

$$H_{k_{I2}} = -r_2x_{10}(k_{P2}\beta_2x_2-k_{P2}x_3+k_{I2}x_{10}) + \frac{1}{T_{g2}}x_{10}\lambda_5 \qquad (4.18)$$

The system canonic equations have been programmed for the computer with system parameters [15]:

$M_1 = 0.04$ $G_1 = 0.01$ $T_{g1} = 0.5$ $T_{t1} = 0.5$ $E_1 = 0.03$

$M_2 = 0.03$ $G_2 = 0.008$ $T_{g2} = 1.2$ $T_{t2} = 0.5$ $D_2 = 0.5$ $E_2 = 0.013$

and performance index weighting coefficients:

$q_i = 1 \quad i = 1,2,\ldots,8$

$r_i = 1 \quad i = 1,2$

In the case that area 1 (steam) is subjected to a different step load change and area 2 (hydro) is disturbance-free, it has been found that the optimal control parameters of the steam area, k_{P1}, k_{I1}, are independent of the magnitude and the sign of the area disturbance. For example, by employing the gradient as determined by equation 1 and 3 of (4.18) over

$[t_0 t_f] = [0, 20 \text{ sec}]$

for a load disturbance of ± 0.005 p.u., conjugate-gradient-descent has been accomplished in 5 iterations (15 trials). The initial guess of the control parameter vector and the step size were taken equal to $\underline{0}$ and 10^{-2} respectively. The optimal gain parameters have been found to be as follows

$k_{P1} = 0.337$

$k_{I1} = 0.513$

and the corresponding cost

$J = 0.01436$

with a gradient norm

$$||H_{K_1}|| = 0.2827 \times 10^{-4}$$

in which

$$K_1^T = (k_{P1}, k_{I1})$$

The augmented dynamic system eigenvalues are given as follows

$$+ 0.01485 \pm j\,1.82, \quad - 3.303 \pm j\,0.097, \quad 2.8668$$

$$- 0.311 \pm j\,0.8418, \text{ and } - 0.641 \pm j\,0.068$$

Consequently, the optimal P-I controller results in a non-asymptotically stable system. However, by obliterating the proportional part and applying the optimization procedure with

$$k_{P1} = 0$$

and

$$H_{k_{P1}} = 0$$

the optimal integral control parameter obtained were

$$k_{I1} = 0.5373$$

and the corresponding cost

$$J = 0.01778$$

with a gradient norm

$$||H_{k_{I1}}|| = 0.1 \times 10^{-5}$$

The augmented dynamic system eigenvalues are given as follows:

$$- 0.04996 \pm j\ 1.751,\ - 0.2194 \pm j\ 7725,\ - 3.368,$$

$$- 0.7229 \pm j\ 0.29145,\ - 3.095, \text{ and } - 2.9016$$

It was found that the optimal control parameter k_{I1} is independent of the load change. Fig. (6.12) shows the system transient and steady state performance in the case of subjecting area 1 to a load change of - 0.005 p.u.

In the case area 2 (hydro) is subjected to variable step load changing and area 1 (steam) regulators are inoperative, it was also found that the hydro area optimal control parameters are independent of the load changes. If area 2 is undergoing, for instance, a load disturbance of ± 0.005 p.u., then by using equation 2 and 4 in (4.18) over a time interval

$$[t_0, t_f] = [0, 20 \text{ sec.}]$$

the conjugate-gradient-descent was completed in 4 iterations (15 trials). The initial guess of the control parameter vector and the step size were taken equal to $\underline{0}$ and 10^{-2} respectively. The optimal control parameters of the hydro area are as follows:

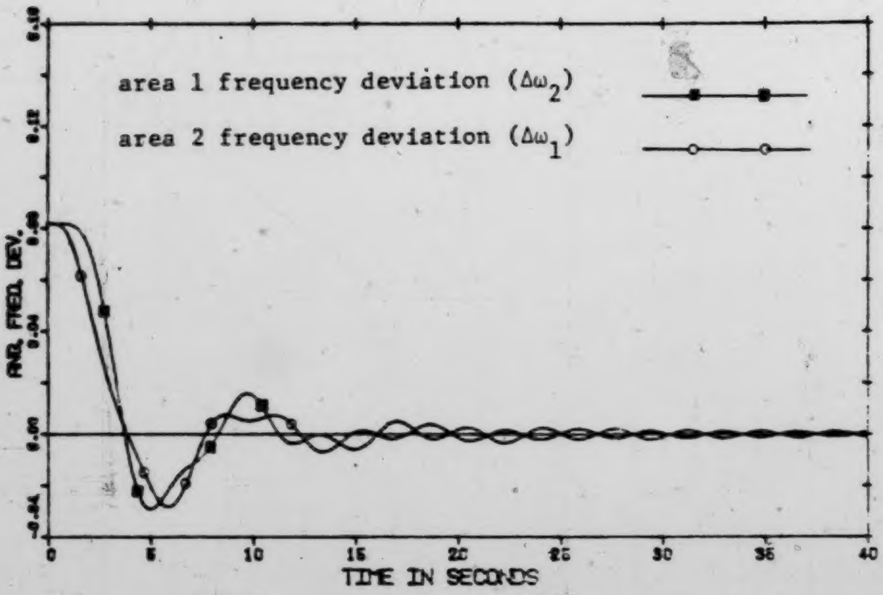

Fig. (6-a) Frequency deviation-time

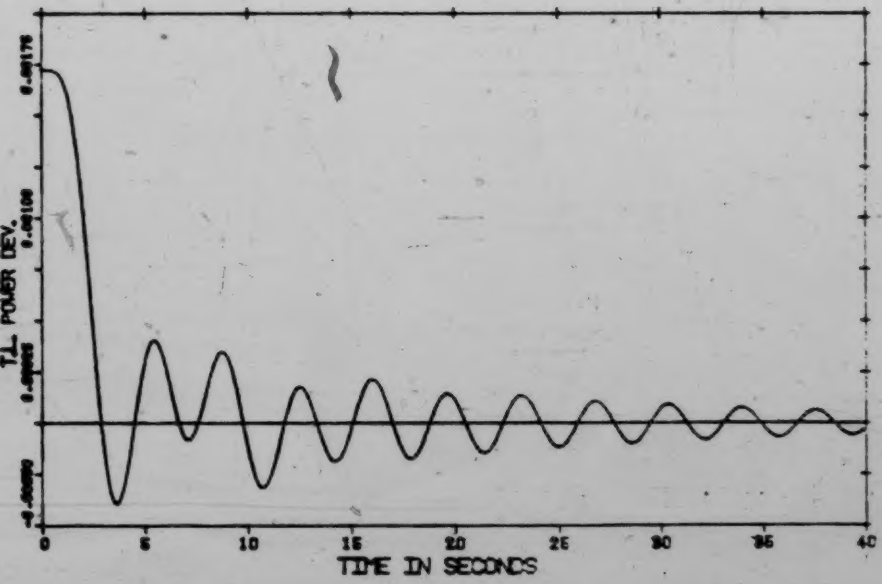

Fig. (6-b) Tie-line power deviation-time

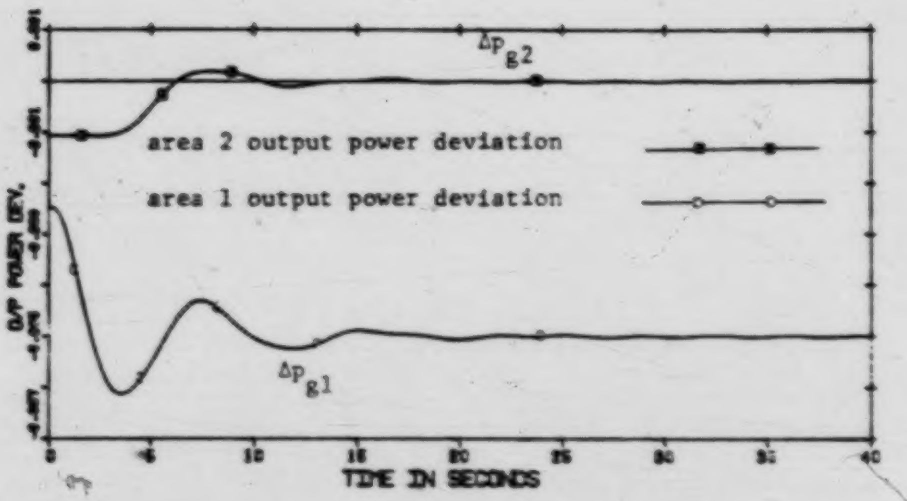

Fig. (6-c) Output power deviation-time

System dynamic response of two mixed area interconnected power system obtained from non-iteractive governor-supplementary regulator,

$\Delta L_1 = -0.005$ p.u., $u_1 = -0.537 \int_{t_0}^{t_f}$

$\Delta L_2 = 0.0$ p.u., $u_2 = 0.0$

$k_{P2} = 0.6629$

$k_{I2} = 0.348$

and the corresponding cost

$J = 0.0206$

with a gradient norm

$||H_{K_2}|| = 0.54 \times 10^{-4}$

in which

$K_2^T = (k_{P2}, k_{I2})$

The system augmented eigenvalues are as follows

$- 3.264 \pm j\, 0.4596, \; - 2.817, \; - 1.086,$

$- 0.3058, \; - 0.2 \pm j.882$, and $- 0.1054 \pm j\, 1.681$

Fig. (7) shows the system transient and steady state performance in the case of area 2 being subjected to - 0.005 p.u. load changing.

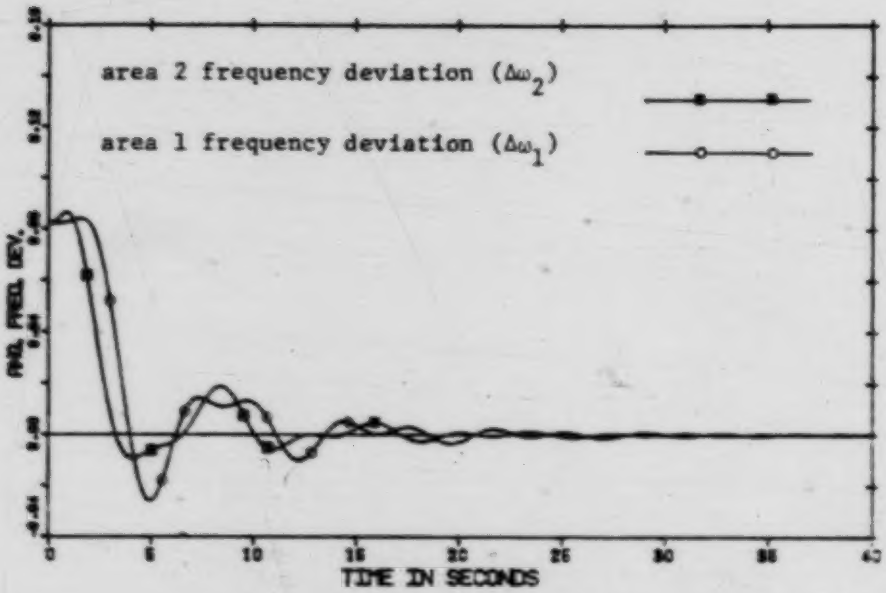

Fig. (7-a) Frequency deviation-time

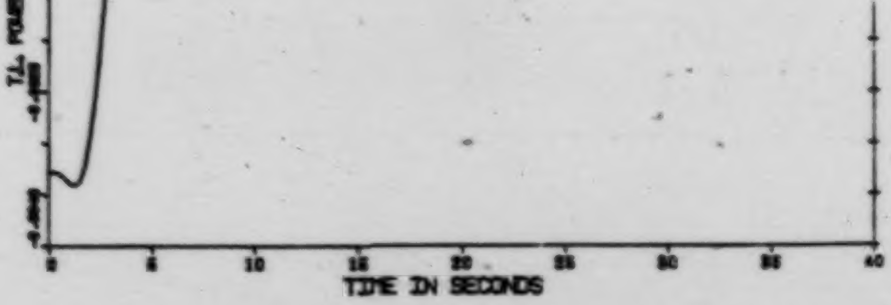

Fig. (7-b) Tie-line power deviation-time

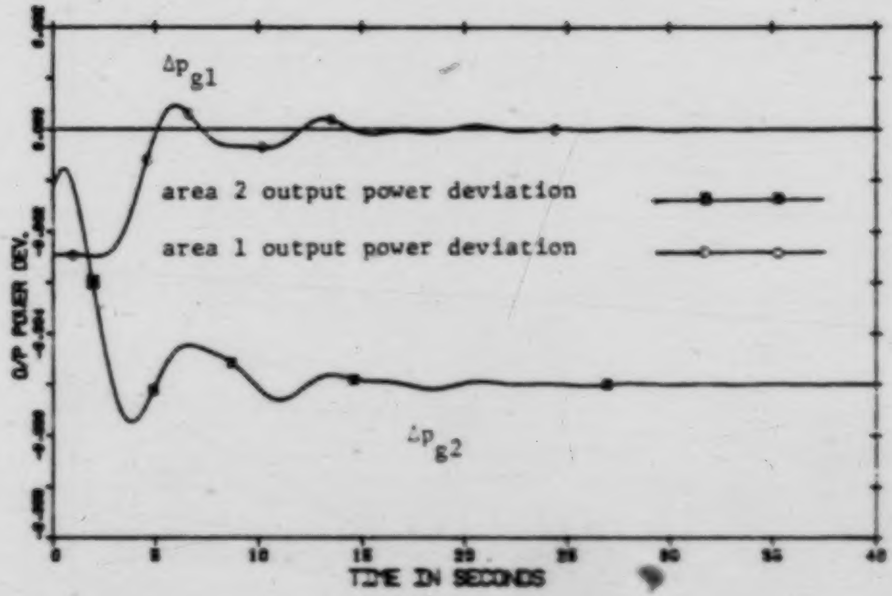

Fig. (7-c) Output power deviation-time

System dynamic response of two mixed area interconnected power system obtained from non-interactive governor-supplementary regulator,

$\Delta L_1 = 0.0$ p.u., $u_1 = 0$

$\Delta L_2 = -0.005$ p.u., $u_2 = 0.6624\, ace_2 - 0.348 \int_{t_0}^{t_f} ace_2 dt$

4.2.2 The optimal interactive governor-supplementary regulator

In this mode of operation, the area's supplementary regulators operate simultaneously with the areas' governors. Accordingly, the problem of achieving the nonintervention principle of the control action cannot be accomplished by choosing the areas frequency bias setting equal to their natural governing characteristics as in the case of the noninteractive governor-supplementary regulator systems, since the area control error of remote areas are not zero. Fortunately, the sign of the frequency and the tie-line power deviations may serve as a criterion to show that whether the areas of an interconnected power system are in need or not to adjust their own generation. If the sign of the frequency and the tie-line power deviations is the same in one area, then this area is being subjected to a local load disturbance and is consequently in need of a control action to accommodate the load change. If not, then; ideally, the area is not locally disturbed and therefore there is no need for its supplementary regulator to take an action to correct the mismatch in the prevailing load-generation in the remote areas. That is,the burden of the system regulating responsibilities are carried by the disturbed area; provided that the disturbed area can fully accommodate its load disturbances.

Basically, the problem of finding the optimal control parameters in the case of employing an interactive governor-supplementary regulator is the same as in the case of the noninteractive one. The system state and co-state equations are the same as in section (4.2.1), except that the initial conditions of the state equations are given as follows:

$$x_i(t_0) = 0 \qquad i = 1,2,\ldots,11 \qquad (4.19)$$

Three cases have been considered in the problem of finding the optimal control parameters of an interactive supplementary regulator.

i A sudden loss or gain of load at one area

When one area, for example, area 1, of the two area interconnected power system is being subjected to a step load change ($\mp \Delta L_1$), it is assumed that the disturbed area supplementary regulator will take all the regulating responsibilities while the remote area supplementary regulator is not operating; that is,

$$\begin{aligned} u_1 &= - k_{P1}\, ace_1 - k_{I1} \int_{t_0}^{t_f} ace_1\, dt \\ u_2 &= 0 \end{aligned} \tag{4.20}$$

in which

$$ace_1 = \Delta P_{t12} + \beta_1\, \Delta\omega_1$$

Two schemes have been employed to design the area 1 supplementary regulator:

(a) Designing an optimal supplementary regulator with frequency bias setting equal to the area natural governing characteristic.

(b) Designing an optimal supplementary regulator with optimal frequency bias setting.

In the first scheme, the bias setting is given by the following identity

$$\beta_1 = E_1 + G_1 \tag{4.21}$$

In this case the control parameter vector is defined by

$$K_1^T = (k_{P1}, k_{I1}) \qquad (4.22)$$

The optimal control parameters were found to be independent of the load changing. For example, in the case that area 1 is being subjected to a load change, ± 0.005 p.u. the conjugate-gradient-descent was completed in 4 iterations (14 trials). The initial guess of the control parameter vector and the step size were taken equal to $\underline{0}$ and 10^{-2} respectively. The optimal control parameters are as follows

$$K_1^T = (0.08335, 0.451) \qquad (4.23)$$

and the corresponding cost

$$J = 0.0242$$

with a gradient norm

$$||H_{K_1}|| = 0.2 \times 10^{-4}$$

In the second scheme, the problem of designing an optimal supplementary regulator with frequency bias-setting is considered. The problem was reduced to a single parameter optimization problem. The control gain parameters k_{P1} and k_{I1} were taken from identity (4.23). In this case the gradient is given as follows

$$H_{\beta_1} = -r_1 k_{P1} x_1 [k_{P1}(\beta_1 x_1 + x_3) + k_{I1} x_9]$$

$$+ \frac{1}{T_{g1}} x_2 \, k_{P1} \, \lambda_4 - x_1 \lambda_9 \qquad (4.24)$$

Adopting the technique which was developed in Chapter 2, it was found that the optimal bias setting is independent of the load changing. In the case of ± 0.005 p.u. load changing, the initial guess of the frequency bias-setting and the initial step size were taken equal to 0.04 (natural governing characteristic) and 0.1 respectively. After 3 iterations (20 trials), the optimal bias setting is given as

$$\beta_1 = 0.03595$$

and the corresponding cost

$$J = 0.02417$$

with a gradient norm

$$||H_{\beta_1}|| = 0.3133 \times 10^{-5}$$

Therefore, the value of the optimum frequency bias-setting (0.036) is less than the value of the area natural governing characteristic (0.04). It was found that the effect of the area bias-setting on the value of the cost functional, the optimality criterion, is negligible. To show that, as well as the effect of the system parameters on the system performance, a sensitivity analysis has been conducted. This will be presented later.

Similarly, the problem of finding the optimal control parameters in the case that area 2 is being subjected to load changing ($\pm \Delta L_2$) and the remote area supplementary regulator is not operating; that is,

$$u_1 = 0$$

$$u_2 = - k_{P2}\, ace_2 - k_{I2} \int_{t_0}^{t_f} ace_2 dt \qquad (4.25)$$

in which

$$ace_2 = -\Delta P_{t12} + \beta_2\, \Delta\omega_2$$

or

$$ace_2 = \Delta P_{t21} + \beta_2\, \Delta\omega_2$$

has been studied.

In the case of employing a tie-line-bias control with frequency bias-setting equal to the area natural governing characteristic; that is,

$$\beta_2 = E_2 + G_2$$

the optimal control parameters of area 2 were found to be

$$K_2^T = (0.607,\ 0.32)$$

in which

$$K_2^T = (k_{P2}, k_{I2})$$

For example, if area 2 is being subjected to a load changing of ± 0.005 p.u., the corresponding cost is given by

$$J = 0.0302$$

with a gradient norm

$$||H_{K_2}|| = 0.38639 \times 10^{-4}$$

The initial control parameter and the initial step size were taken to be $\underline{0}$ and 10^{-1} respectively.

In the case of designing the area 2 tie-line bias controller with optimum bias-setting, the gradient of the Hamiltonian with respect to the bias setting is given by

$$H_{\beta_2} = - r_2 k_{P2} x_2 [k_{P2}(\beta_2 x_2 - x_3) + k_{I2} x_{10}] + \frac{1}{T_{g2}} x_2 k_{P2} \lambda_5 - x_2 \lambda_{10} \tag{4.26}$$

Using the gradient as determined in equation (4.26) with the control gain parameters given as

$$K_2^T = (0.607, 0.32)$$

over

$[t_0, t_f]$

the conjugate-gradient descent was accomplished with the program described in Chapter 2. The initial guess of the frequency bias-setting was taken equal to 2 natural governing characteristic (0.021). After 7 iterations (10 trials), the optimal frequency bias-setting was found to be

$$\beta_2 = 0.0148$$

with a corresponding cost corresponding to ± 0.005 p.u., load disturbance

$$J = 0.029985$$

and a gradient norm

$$||H_{\beta_2}|| = 0.141 \times 10^{-15}$$

Once more, it can be seen that the effect of employing a tie-line bias controller with optimum frequency bias setting produces a negligible effect in the system optimality criterion, J. For example, in the case of ± 0.005 p.u., load change, the loss of optimality, ε,

$$\varepsilon = \{[J_{opt.\beta_2} - J_{non\ opt.\beta_2}] / J_{non\ opt.\beta_2}\} \times 100$$

is equal to 0.712 percent.

Fig. (8) shows the system performance when area 1 is being subjected to -0.005 p.u., load change and the area 2 supplementary regulator is not

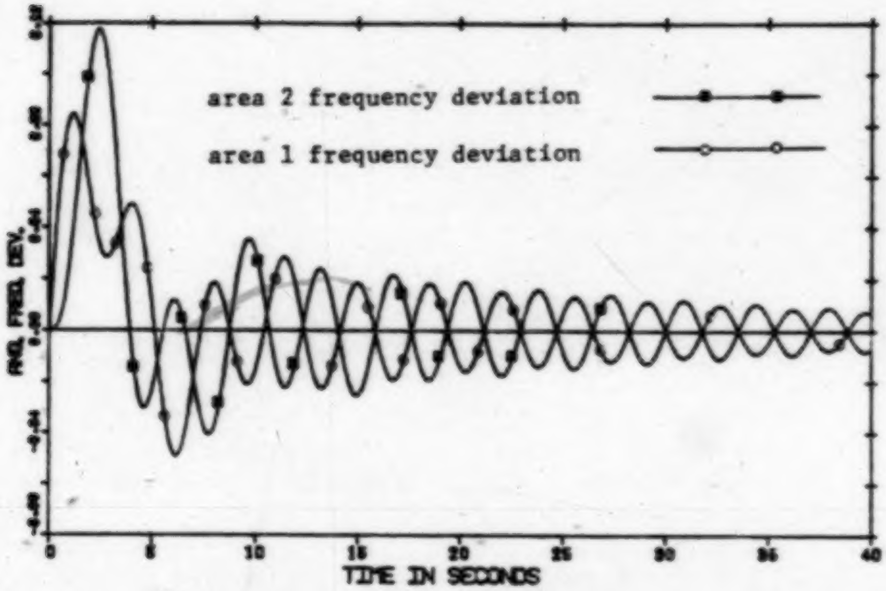

Fig. (8-a) Frequency deviation-time

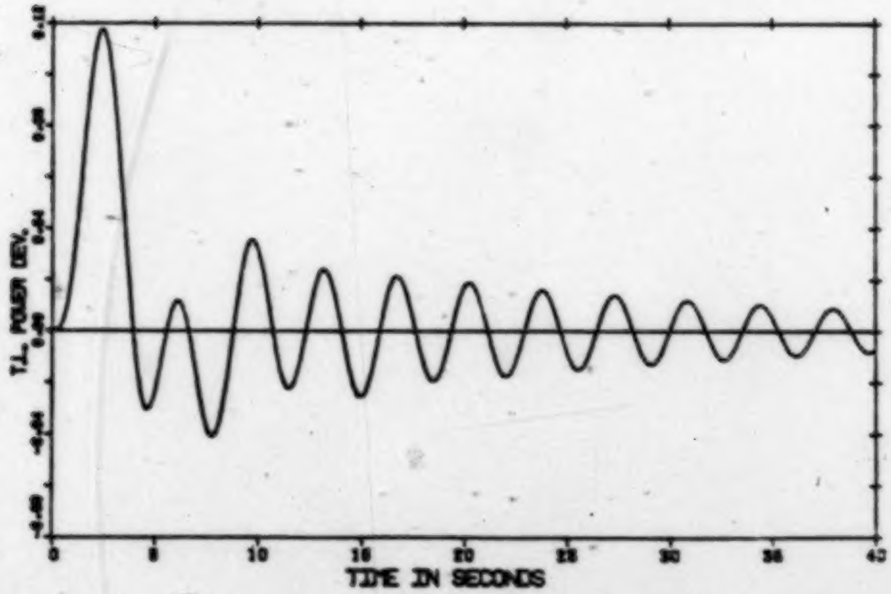

Fig. (8-b) Tie-line power deviation

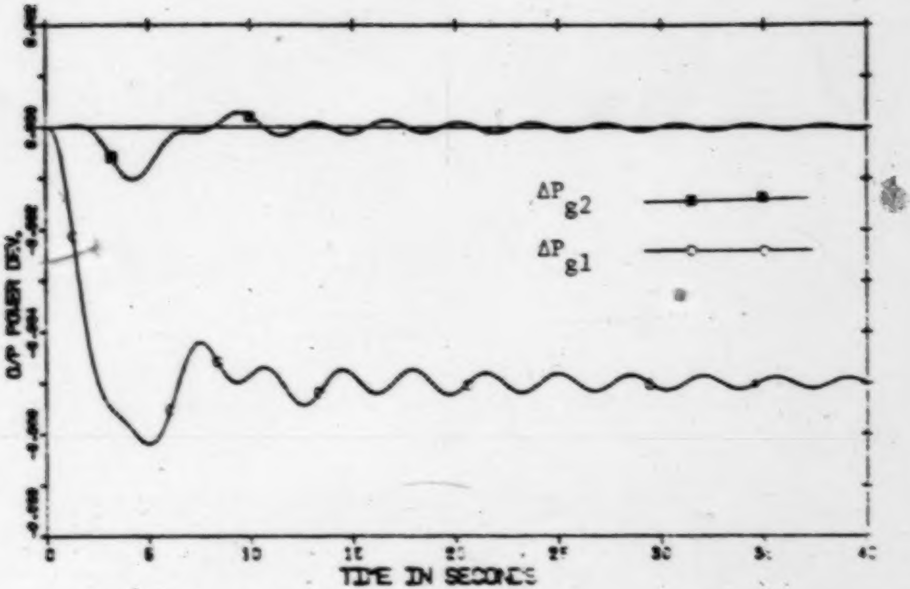

Fig. (8-c) Output power deviation-time

System dynamic response of two mixed area system obtained from interactive governor-supplementary regulator,

$\Delta L_1 = -\ 0.005$ p.u., $u_1 = -0.0834\ ace_1 - 0.451 \int_{t_0}^{t_f} ace_1 dt$

$\Delta L_2 = 0.0$, $u_2 = 0$

operating. Fig. (9) shows the same when area 2 is being subjected to a - 0.005 p.u., load change and the area 1 supplementary regulator is not operating.

ii Loss of a tie-line between two areas

This is equivalent to a sudden load increase in one area and a sudden load decrease in the other. The system parameters before and after the removal of the tie-line are the same in the case of an d.c. line and not the same in the case that the line is a.c.

In this case, both areas' supplementary regulator should come into action simultaneously. Accordingly, the control law of the IMAIPS can be written as follows

$$\begin{aligned} u_1 &= - k_{P1}\, ace_1 - k_{I1} \int_{t_0}^{t_f} ace_1 dt \\ u_2 &= - k_{P2}\, ace_2 - k_{I2} \int_{t_0}^{t_f} ace_2 dt \end{aligned} \qquad (4.27)$$

in which

$$\begin{aligned} ace_1 &= \Delta P_{t12} + \beta_1 \Delta\omega_1 \\ ace_2 &= \Delta P_{t21} + \beta_2\, \Delta\omega_2 \end{aligned} \qquad (4.28)$$

For example, if the system is subjected to a load disturbance [8], [15]

$\Delta L_1 = - 0.005$

$\Delta L_2 = + 0.005$

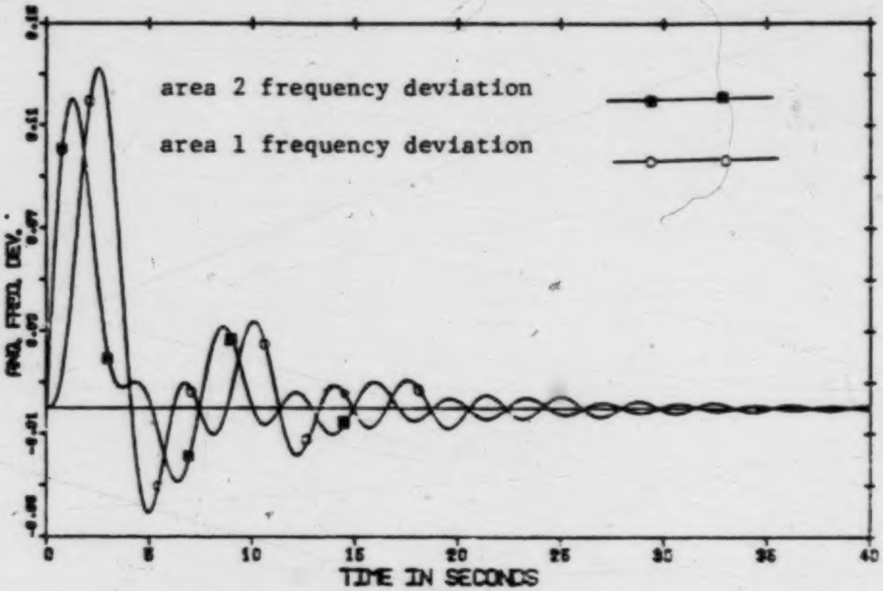

Fig. (9-a) Frequency deviation-time

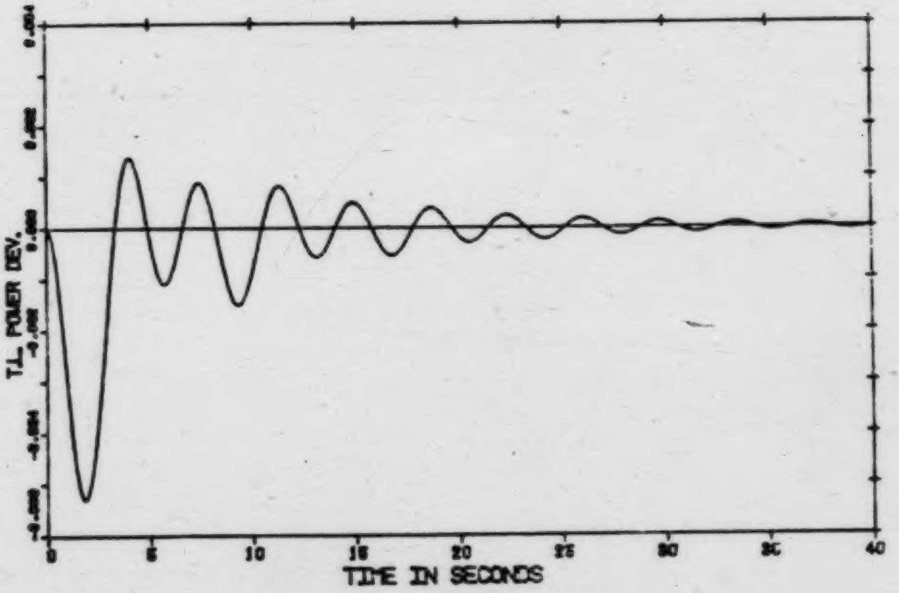

Fig. (9-b) Tie-line power deviation

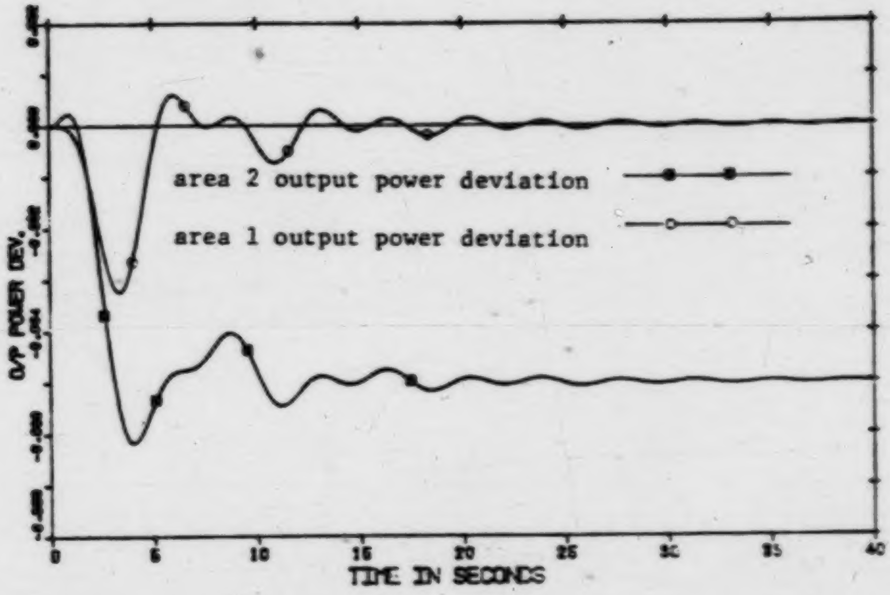

Fig. (9-c) Output power deviation

System dynamic response of two mixed area system obtained from interactive governor-supplementary regulator,

$\Delta L_1 = 0.0$, $u_1 = 0$

$\Delta L_2 = -0.005$ p.u., $u_2 = -.706\, ace_2 - 0.32 \int_{t_0}^{t_f} ace_2 dt$

which may represent a sustained removal of one of the tie-lines connecting area 1 to area 2.

The solution of the canonic equations given by equations (4.16), (4.17) and (4.18) subject to the boundary conditions

$$x_i(t_0) = 0 \qquad i = 1,2,\ldots, 12$$

yields the control parameter vector

$$K = \text{col.}\ (k_{P1}, k_{P2}, k_{I1}, k_{I2}) \tag{4.29}$$

By using the technique developed in Chapter 2, the optimum value of the control parameter vector was found to be

$$K = \text{col.}\ (-.4484, -.0025, .3472, .442)$$

and the value of the optimum cost function

$$J = 0.0192$$

with a gradient norm

$$||H_K|| = 0.971 \times 10^{-4}$$

The initial guess of the control parameter and the initial step size have been taken equal to $\underline{0}$ and 0.1 respectively.

In the case of employing an integral control action; that is,

$$K = \text{col.}\ (k_{I1},\ k_{I2}) \tag{4.30}$$

the optimal control parameters were found to be

$$K = \text{col.}\ (0.255,\ 0.27)$$

and the value of the optimum cost function

$$J = 0.0276$$

In the case of applying the control parameters obtained in [15] which are given as follows

$$k_{I1} = 0.09$$

$$k_{I2} = 0.4$$

The corresponding cost

$$J = 0.06946$$

which is almost 2.5 times as the proposed controller. Figure (10) shows a comparison between the system performance in both cases.

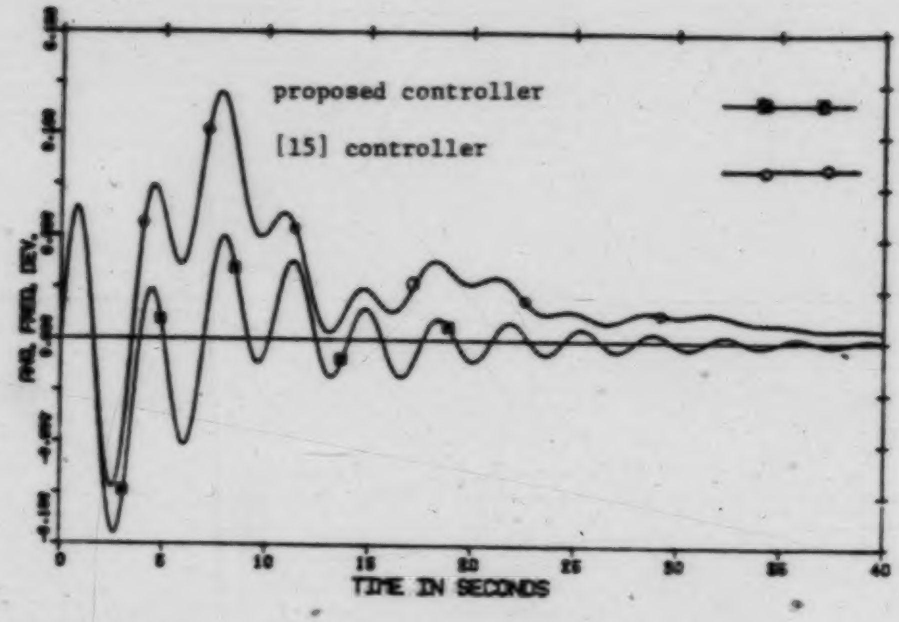

Fig. (10-a) Area 1 frequency deviation-time

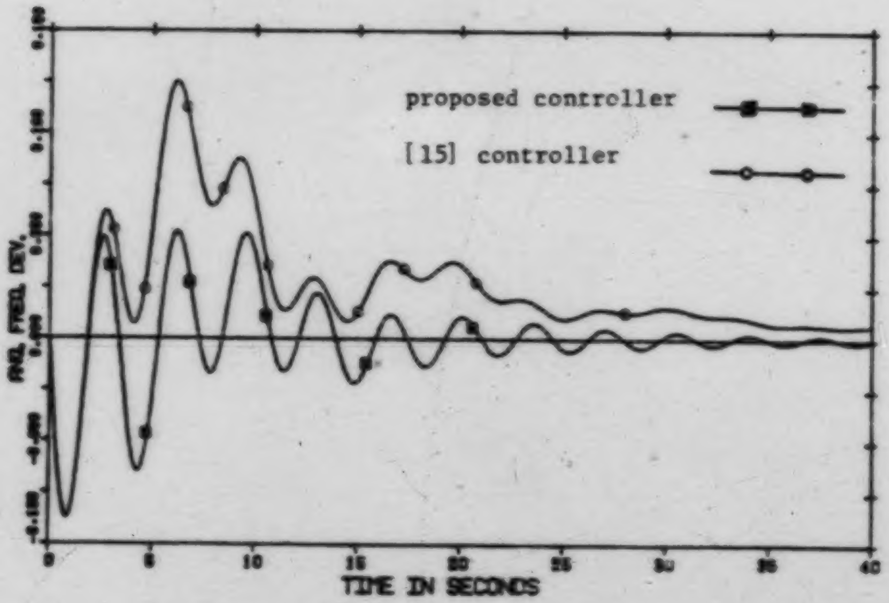

Fig. (10-b) Area 2 frequency deviation-time

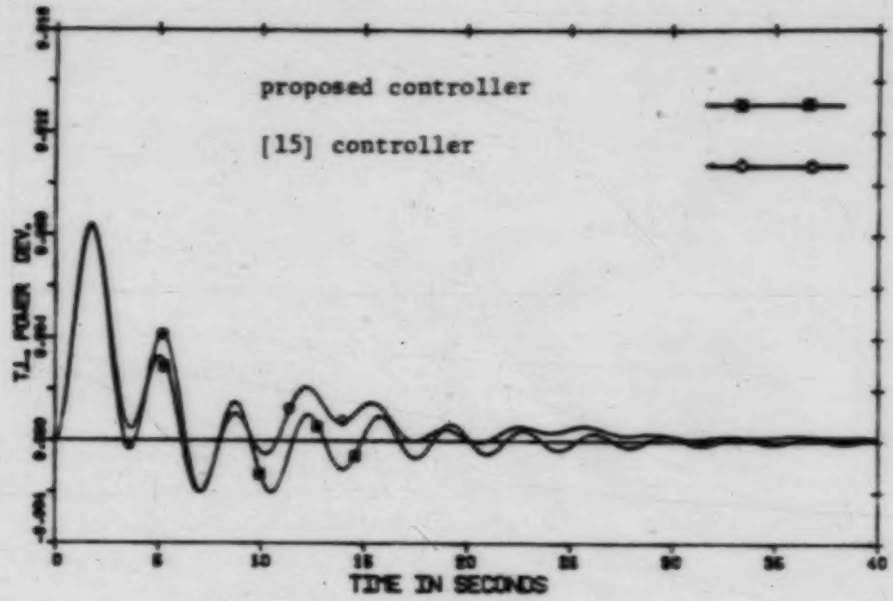

Fig. (10-c) Tie-line power deviation-time

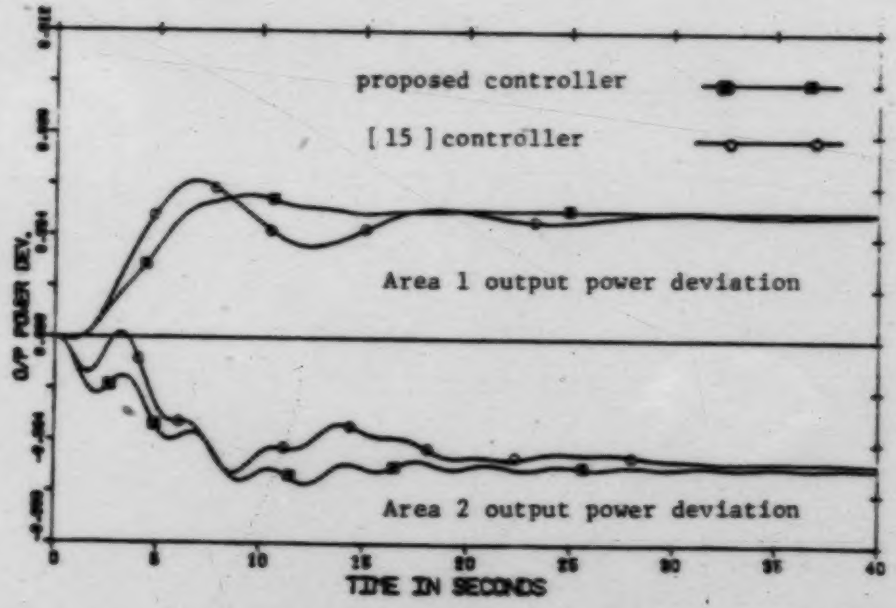

Fig. (10-d) Output power deviation-time

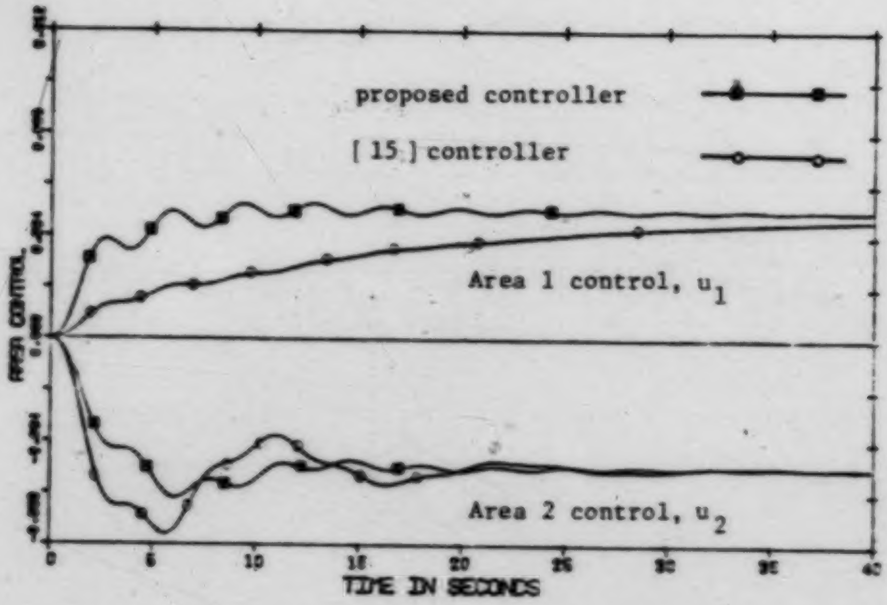

Fig. (10-e) Areas' control-time

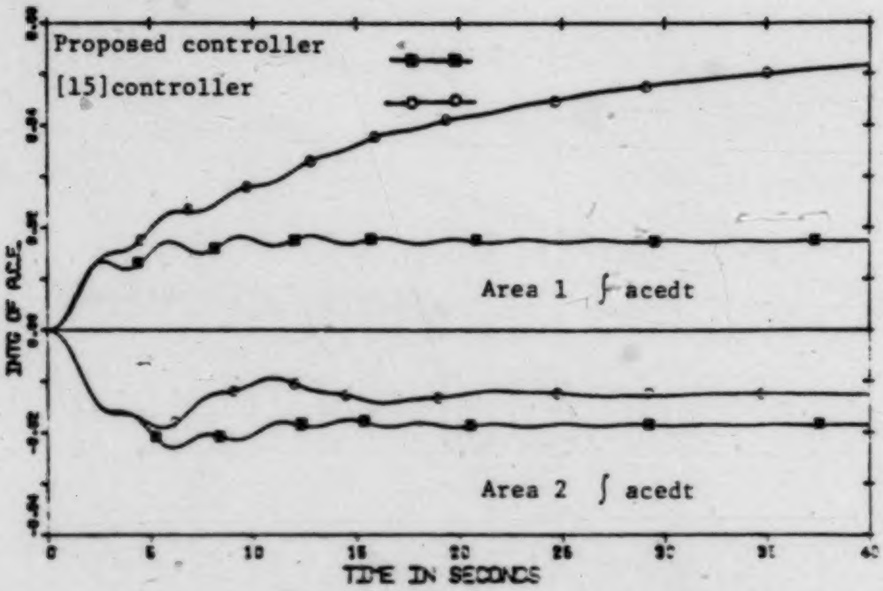

Fig. (10-f) ∫ acedt-time

System dynamic response of two mixed area system obtained from the proposed controller

$u_1 = - 0.0255 \int ace_1 dt$

$u_2 = - 0.27 \int ace_2 dt$

and the controller advised in [15]

$u_1 = -0.09 \int ace_1 dt$

$u_2 = -0.4 \int ace_2 dt$

$\Delta L_1 = - 0.05$ p.u., $\Delta L_2 = + 0.005$ p.u.

iii A sudden loss of generation at one area

This is equivalent to case I except that the system parameters before and after the disturbance are not the same. The value of the disturbed area inertia constant is lower after the cccurrence of the disturbance for instance, if area 1 undergoes loss of $-\Delta L_1$ p.u., of its generation, the new value of M_1 is

$$M_1^n = M_1 (1 - \Delta L_1) \tag{4.31}$$

in which

M_1^n = area 1 inertia constant after the occurrence of the disturbance

M_1 = area 1 inertia constant before the occurrence of the disturbance

It was found that values of the optimal control parameters depend upon the values of the areas' inertia constant. Table (1) shows the relationship between area 1 inertia constant and the optimal control parameters. Table (2) shows the same for area 2. However, the optimal control parameters which were obtained in the case when considering that the areas' inertia are constant results in a stable system with a slight loss of optimality compared to the one in which the areas' inertia were considered variable. For example, in the case that area 1 is being subjected to a 0.1 p.u. loss of its generation and area 1 is employing a supplementary regulator with gain parameters

$k_{P1} = 0.0833$

$k_{I1} = 0.451$

Generation loss	Inertia constant	k_{P1}	k_{I1}
-0.005	0.0398	0.0833	0.4510
-0.01	0.0396	0.0823	0.4511
-0.02	0.0392	0.0792	0.4513
-0.04	0.0384	0.0755	0.4514
-0.06	0.0376	0.0721	0.4512
-0.08	0.0368	0.0694	0.4506
-0.1	0.036	0.0673	0.450

Table (1)

Steam area optimum control parameters

versus the area inertia

Generation loss	Inertia constant	k_{P2}	k_{I2}
-0.005	0.02985	0.605	0.321
-0.01	0.0297	0.603	0.321
-0.02	0.0294	0.500	0.322
-0.04	0.0288	0.592	0.324
-0.06	0.0282	0.585	0.326
-0.08	0.0276	0.57	0.328
-0.10	0.027	0.57	0.33

Table (2)

Hydro area optimum control parameters

versus the area inertia

which resulted from solving the system canonic equations with the assumption that the system parameters before and after the disturbance are the same; that is

$$M_1^n = M_1 = 0.04$$

this result in the cost function

$$J_1 = 0.099361 \times 10^{+1}$$

when it is applied to the system with $M_1^n = 0.036$. In the case that the system parameters before and after are not considered the same; that is,

$$M_1 = 0.04$$

$$M_1^n = 0.036$$

The optimal control parameters are given as

$$k_{P1} = 0.0673$$

$$k_{I1} = 0.45$$

and the value of the optimum cost function

$$J_2 = 0.9928 \times 10^{+1}$$

Hence, in the case of employing a control strategy which neglects the changes in the systems' inertia results in a loss of optimality

$$\varepsilon = \frac{J_1-J_2}{J_1} \times 100 = 0.0815 \text{ percent}$$

Therefore, the assumption that the systems' inertia are equal before and after a generation disturbance is satisfactory.

4.3 Sensitivity analysis of the LFC problem

Sensitivity considerations of dynamical systems are very important in designing control systems. In designing a control system one must first develop a mathematical model which represents the physical behavior of these systems. There is always a discrepancy between the physical reality and the mathematical model. This is due to; inexact identification of the systems parameters, unpredictable changes in the systems environment, etc.

Sensitivity analysis is of a particular importance in the case of employing modern control theory to design systems with prescribed behavior. The results obtained employing modern control theory are useless in practice, if the mathematical model used,prove to be very sensitive to parameter variations. Therefore, it is very important to perform sensitivity analysis whenever modern control theory is adopted.

A sensitivity analysis has been performed on the problem of LFC of two mixed area interconnected power system to study the system performance to the system parameters as well as to the control parameter variations. The sensitivity analysis of a dynamic system can be defined as follows:

Given a dynamic system described by the following ordinary differential equation

$$\dot{X}_1 = A_1(K)X_1+\Gamma_1\Delta L, X_1 \; (t_0) = 0 \tag{4.32}$$

in which

X_1 is an nxl state vector

K is an mxl parameter vector

the problem posed is to find the sensitivity of the system to the parameter variations, ΔK. One possible way to find the system sensitivity to the parameter variation ΔK is to find the sensitivity of a cost functional

$$J = \frac{1}{2} \int_{t_0}^{t_f} X_1^T Q_1 X_1 dt \tag{4.33}$$

with respect to a variation in the parameter vector ΔK.

To handle this problem, one can introduce a new variable

$$\dot{x}_0 = X_1^T Q_1 X_1 \qquad \dot{x}_0(t_0) = 0 \tag{4.34}$$

and by employing the trajectory sensitivity analysis of dynamic systems one can find the sensitivity of $x_0(t_f)$ with respect to the parameter K.

Then, define a new variable X as follows

$$X = \text{col.}\ (X_1,\ x_o) \tag{4.35}$$

The augmented dynamic system can be written as

$$\dot{X} = A(K)X + \Gamma L, \quad X(t_0) = 0 \tag{4.36}$$

By taking the partial derivative of equation (4.36) with respect to k_i, one obtains

$$\frac{\partial \dot{X}}{\partial k_i} = \frac{\partial A(K)}{\partial X} \frac{\partial X}{\partial k_i} + \frac{\partial A(K)X}{\partial k_i}, \quad \frac{\partial X(t_0)}{\partial k_i} = 0 \quad i = 1,2,\ldots m$$

or

$$\dot{S}_i = \frac{\partial A(K)}{\partial X} S_i + \frac{\partial A(K)X}{\partial k_i}, \quad S_i(t_0) = 0, \; i = 1,2,\ldots,m \tag{4.37}$$

in which

$S_i(t)$ is an $[(n+1)\mathrm{x}1]$ sensitivity vector, $t \in [t_0, t_f]$

Therefore, the cost functional sensitivity with respect to the elements of K is given as

$$\frac{\partial J}{\partial k_i} = S_i(t_f) \quad i = 1,2,\ldots,m \tag{4.38}$$

Equation (4.38) can be easily calculated by the forward integration of equation (4.36) and equation (4.37).

In the case that area 1 is being subjected to -0.005 p.u. load change, and the nonintervention principle of the areas supplementary regulator is satisfied, the cost functional sensitivity with respect to area 1 control parameter variations is as follows

$$S_{k_{P1}} = -.235 \times 10^{-3}$$

$$S_{k_{I1}} = -.426 \times 10^{-2}$$

$$S_{\beta_1} = -0.3883 \times 10^{-3}$$

For a 0.05 change in the control parameters k_{P1}, k_{I1}, and β_1, the percentage change in the value of the cost function is - 0.004, - 0.397, and - 0.003 respectively. This will emphasize the effect of the integral control parameter on the system performance.

The sensitivity of the cost function with respect to the area 1 parameter variations are

$$S_{T_{g1}} = -.469 \times 10^{-2}$$

$$S_{T_{t1}} = +.145 \times 10^{-3}$$

$$S_{E_1} = -.415 \times 10^{-2}$$

For example, 5 percent change in T_{g1}, T_{t1}, and E_1 results in - 0.484, + 0.015, and - 0.026 percentage change in the value of the cost function. This shows the prominent effect of the governor time constant on the system performance compared to that of the turbine time constant and the speed regulation. Therefore, more consideration should be given to the process of identifying the areas' governor time constant than to that of the other two parameters.

Similar results were obtained from area 2 sensitivity analysis. For example, when area 2 is being subjected to - 0.005 p.u. load change, the cost functional sensitivity with respect to area control parameters were found to be

$$S_{k_{P2}} = - 0.723 \times 10^{-2}$$

$$S_{k_{I2}} = - 0.10 \times 10^{-1}$$

$$S_{\beta_2} = - 0.733 \times 10^{-2}$$

e.g. for a 5 percent change in the area control parameters k_{P2}, k_{I2}, and β_2, there is a 0.0725, 0.532, and 0.0255 percentage change in the value of the cost functional.

The sensitivity of the cost functional with respect to area 2 parameter variations are as follows

$$S_{T_{g2}} = - 0.281 \times 10^{-2}$$

$$S_{T_{t2}} = - 0.25 \times 10^{-3}$$

$$S_{D_2} = - 0.421 \times 10^{-3}$$

$$S_{E_2} = - 0.12115 \times 10^{-1}$$

which again show that the area 2 governor time constant has a conspicuous influence on the system performance.

4.4 Nonlinear load frequency control of interconnected power system

The problem of load frequency control (LFC) exhibits many nonlinearities. The most significant of these arise from the tie-lines connecting the areas of an interconnected system, and the dead-band of the governors. The former is overriding when the system is subjected to large disturbances which produce large power angle deviations while the latter is prominent for small disturbances. In this section, the tie-line nonlinearity is considered.

The tie-line nonlinearity arises from the relation between the tie-line power and the power angle deviation. The tie-line power deviation of area i of an n-area interconnected power system is given by

$$\Delta P_{ti} = \sum_{\substack{j=1 \\ j\neq i}}^{n} T_{ij} \sin \delta_{ij}^{0}(\cos\Delta\delta_{ij}-1) + T_{ij}\cos\delta_{ij}^{0}\sin\Delta\delta_{ij} \tag{4.39}$$

Miniesy and Bohn [16] have proposed a two level controller for the problem of nonlinear load frequency control. The first level is a local feedback control for each area of the interconnected system. This control is a function of its own area state variables. The second level is an intervention open loop which is utilized to compensate for neglecting the coupling state variable between the areas of the interconnected system, and the nonlinearities due to the tie-lines. The idea of employing a multi-level control strategy in the area of load frequency control is not adequate. First, it increases the controller design complexity. Second; it is not used in practice. Doraiswami [17] has considered the problem of LFC with a tie-line nonlinearity from the stochastic viewpoint. An observer was employed to implement the control law. Once an observer is introduced into the system, the cost increases and the control is no longer optimal. Therefore, it is necessary to design a one level optimal controller,and the control law must only be a function of the measurable states.

4.4.1 The mathematical model

The dynamic nonlinear model of steam-hydro interconnected power system (SHIPS), is the same as was considered in the preceeding sections, except that the state variable x_3; which was representing the tie-line power deviation, is representing the power angle deviation. Therefore, the dynamic model of SHIPS taking into account the tie-line nonlinearity, is given by the following system of equations

$$
\begin{aligned}
\dot{x}_1 &= -\frac{G_1}{M_1} x_1 - \frac{1}{M_1} (A\cos x_3 + B\sin x_3 - A) + \frac{1}{M_1} x_6 - \frac{\Delta L_1}{M_1} \\
\dot{x}_2 &= -\frac{G_2}{M_2} x_2 + \frac{1}{M_2} (A\cos x_3 + B\sin x_3 - A) + \frac{1}{M_2} x_8 - \frac{\Delta L_2}{M_2} \\
\dot{x}_3 &= x_1 - x_2 \\
\dot{x}_4 &= -\frac{E_1}{T_{g1}} x_1 - \frac{1}{T_{g1}} x_4 + \frac{1}{T_{g1}} u_1 \\
\dot{x}_5 &= -\frac{E_2}{T_{g2}} - \frac{1}{T_{g2}} x_5 + \frac{1}{T_{g2}} u_2 \\
\dot{x}_6 &= \frac{1}{T_{t1}} x_4 - \frac{1}{T_{t1}} x_6 \\
\dot{x}_7 &= \frac{1}{T_{t2}} x_5 - \frac{1}{T_{t2}} x_7 \\
\dot{x}_8 &= -\frac{2}{T_{t2}} x_5 + \frac{2}{(T_{t2} + D_2)} x_7 - \frac{2}{D_2} x_8
\end{aligned}
\tag{4.40}
$$

in which

$$A = T_{12} \sin \delta_{12}^{0}$$

$$B = T_{12} \cos \delta_{12}^{0}$$

The control law of each area is specified to be proportional-plus-integral of the area control error. In terms of the systems' state variable, u_1 and u_2 can be written as

$$u_1 = - k_{P1}(\beta_1 x_1 + A\cos x_3 + \beta \sin x_3 - A) - k_{I1} \int_{t_0}^{t_f} (\beta_1 x_1 + A\cos x_3 + \beta \sin x_3 - A)dt$$

$$u_2 = - k_{P2}(\beta_2 x_2 - A\cos x_3 - \beta \sin x_3 + A) \int_{t_0}^{t_f} (\beta_2 x_2 - A\cos x_3 - \beta \sin x_3 + A)dt \qquad (4.41)$$

The problem posed is to find the optimal controls u_1 and u_2 or equivalently the optimal parameters k_{P1}, k_{P2}, k_{I1} and k_{I2} which minimize the cost functional

$$J = \frac{1}{2} \int_{t_0}^{t_f} \Big(\sum_{\substack{i=1 \\ i \neq 3}}^{8} q_i x_i^2 + q_T(A\cos x_3 + \beta \sin x_3 - A) + \sum_{i=1}^{2} r_i u_i^2 \Big)dt \qquad (4.42)$$

subject to the constraints given by equation(4.41) and equation (4.42). Also, the steam rate of generation should be less or equal Ψ p.u. mw/sec.

To cast the system dynamic equations into the familiar state space form, one might introduce new variables x_9 and x_{10} such that

$$\dot{x}_9 = \beta_1 x_1 + A\cos x_3 + B\sin x_3 - A \qquad (4.43)$$

and

$$\dot{x}_{10} = \beta_2 x_2 - A\cos x_3 - B\sin x_3 + A$$

to formulate the control law into the proportional form. Consequently, u_1 and u_2 can be written as follows

$$u_1 = -k_{P1}(\beta_1 x_1 + A\cos x_3 + B\sin x_3 - A) - k_{I1} x_9$$

$$u_2 = -k_{P2}(\beta_2 x_2 - A\cos x_3 - B\sin x_3 + A) - k_{I2}\, x_{10} \qquad (4.44)$$

The inequality constraints due to the steam area rate of generation limit can be expressed mathematically as follows

$$\frac{1}{T_{t1}} x_4 - \frac{1}{T_{t1}} x_6 \leq \psi \qquad (4.45)$$

To convert equation (4.45) into an equality constraint, another new variable x_{11} is introduced

$$\dot{x}_{11} = (\frac{1}{T_{t1}} x_4 - \frac{1}{T_{t1}} x_6 - \psi)\ H_1(\frac{1}{T_{t1}} x_4 - \frac{1}{T_{t1}} x_6 - \psi) \qquad (4.46)$$

where

$$H_1(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 0 \\ 1 & \text{if } \alpha > 0 \end{cases}$$

The problem of finding the optimal control parameters (k_{P1}, k_{P2}, k_{I1}, and k_{I2}) of the dynamic system; given by equation (4.41), equation (4.42), equation (4.44) and equation (4.47), using the technique which has been introduced in Chapter 2 failed to converge. Therefore, another approach was adopted by reformulating the system dynamics into a quadratic form. This was done by introducing a nonlinear transformation [18], on the angular frequency deviation (x_3). This nonlinear transformation is defined by

$$x_{12} = \sin x_3 \qquad (4.47)$$

and, then, define a pseduo-variable, x_{13} by

$$x_{13} = \sqrt{1 - x_{12}^2} \qquad (4.48)$$

For convenient notation, a new variable y is introduced such that

$y_i = x_i \qquad i = 1,2$

$y_j = x_{j+1} \qquad j = 3,4,5,\ldots,10$

then, the system dynamic equations can be written as follows

$$\dot{y}_1 = -\frac{G_1}{M_1} y_1 - \frac{1}{M_1} (A\ y_{12} + B\ y_{11} - A) + \frac{1}{M_1} y_5 - \frac{1}{M_1} \Delta L_1$$

$$\dot{y}_2 = -\frac{G_2}{M_2} y_2 + \frac{1}{M_2} (A\ y_{12} + B\ y_{11} - A) + \frac{1}{M_2} y_7 - \frac{1}{M_2} \Delta L_2$$

$$\dot{y}_3 = -\frac{E_1}{T_{g1}} y_1 - \frac{1}{T_{g1}} y_3 + \frac{1}{T_{g1}} u_1$$

$$\dot{y}_4 = -\frac{E_2}{T_{2}} y_2 - \frac{1}{T_{g2}} y_4 + \frac{1}{T_{g2}} u_2$$

$$\dot{y}_5 = \frac{1}{T_{t1}} y_3 - \frac{1}{T_{t1}} y_5$$

$$\dot{y}_6 = \frac{1}{T_{t2}} y_4 - \frac{1}{T_{t2}} y_6$$

$$\dot{y}_7 = -\frac{2}{T_{t2}} y_4 + \frac{2}{(T_{t2} + D_2)} y_6 - \frac{2}{D_2} y_7$$

$$\dot{y}_8 = \beta_1 y_1 + (A\ y_{12} + B\ y_{11} - A)$$

$$\dot{y}_9 = \beta_2 y_2 - (A\ y_{12} + B\ y_{11} - A)$$

$$\dot{y}_{10} = (\frac{1}{T_{t1}} y_3 - \frac{1}{T_{t1}} y_5 - \psi)^2\ H_1 (\frac{1}{T_{t1}} y_3 - \frac{1}{T_{t1}} y_5 - \psi)$$

$$\dot{y}_{11} = y_{12}(y_1 - y_2)$$

$$\dot{y}_{12} = y_{11}(y_2 - y_1) \qquad (4.49)$$

and

$$u_1 = -k_{P1}(\beta_1 y_1 + A\ y_{12} + B\ y_{11} - A) - k_{I1} y_8 \qquad (4.50)$$

$$u_2 = -k_{P2}(\beta_2 y_2 - A\ y_{12} - B\ y_{11} + A) - k_{I2} y_9$$

To handle a criterion of the form (4.43), one can augment the system differential equations by an additional equation as follows

$$\begin{aligned}\dot{y}_{13} = \frac{1}{2}\{ &\sum_{i=1}^{9} q_i y_i^2 + q_T(A\ y_{12} + B\ y_{11} - A)^2 \\ &+[k_{P1}(\beta_1 y_1 + A\ y_{12} + B\ y_{11} - A) + k_{11} y_8]^2 \\ &+[k_{P2}\ (\beta_2 y_2 - A\ y_{12} - B\ y_{11} + A)^2 + k_{12} y_9]^2\} \qquad (4.51)\end{aligned}$$

then, by defining a Hamiltonian

$$H = \sum_{i=1}^{13} \lambda_i \dot{y}_i$$

and applying Pontryagin's minimum principle, one can get the system co-state equations

$$\begin{aligned}\dot{\lambda}_1 = &- q_1 y_1 - r_1 \beta_1 k_{P1}\{k_{P1}(\beta_1 y_1 + A\ y_{12} + B\ v_{11} - A) + k_{I1} y_8\} \\ &+ \frac{G_1}{M_1}\lambda_1 + (\frac{E_1}{T_{g1}} + \frac{\beta_1}{T_{g1}} k_{P1})\ \lambda_3 - \beta_1 \lambda_8 - y_{12} \lambda_{11} + y_{11}\ \lambda_{12}\end{aligned}$$

$$\dot{\lambda}_2 = -\,q_2y_2 - r_2\beta_2k_{P2}\{k_{P2}(\beta_2y_2 - A\,y_{12} - B\,y_{11} + A) + k_{I2}y_9\}$$

$$+ \frac{G_2}{M_2}\,\lambda_2 + (\frac{E_2}{T_{g2}} + \frac{\beta_2}{T_{g2}}\,k_{P2})\,\lambda_4 - \beta_2\lambda_9 + y_{12}\lambda_{11} - y_{11}\lambda_{12}$$

$$\dot{\lambda}_3 = -\,q_3y_3 + \frac{1}{T_{g1}}\,\lambda_3 - \frac{1}{T_{t1}}\,\lambda_5 - \frac{2}{T_{t1}}\,(\frac{1}{T_{t1}}\,y_3 - \frac{1}{T_{t1}}\,y_5 - \psi)H_1\lambda_{10}$$

$$\dot{\lambda}_4 = -\,q_4y_4 + \frac{1}{T_{g2}}\,\lambda_4 - \frac{1}{T_{t2}}\,\lambda_6 + \frac{2}{T_{t2}}\,\lambda_7$$

$$\dot{\lambda}_5 = -\,q_5y_5 - \frac{1}{M_1}\,\lambda_1 + \frac{1}{T_{t1}}\,\lambda_5 + \frac{2}{T_{t1}}\,(\frac{1}{T_{t1}}\,y_3 - \frac{1}{T_{t1}}\,y_5 - \psi)H_1\lambda_{10}$$

$$\dot{\lambda}_6 = -\,q_6y_6 + \frac{1}{T_{t2}}\,\lambda_6 - \frac{2}{T_{t2} + D_2}\,\lambda_7$$

$$\dot{\lambda}_7 = -\,q_7y_7 - \frac{1}{M_2}\,\lambda_2 + \frac{2}{D_2}\,\lambda_7$$

$$\dot{\lambda}_8 = -\,q_8y_8 - r_1k_{I1}\{k_{P1}(\beta_1y_1 + A\,y_{12} + B\,y_{12} - A) + k_{I1}y_8\}$$

$$+ \frac{1}{T_{g1}}\,k_{I1}\,\lambda_3$$

$$\dot{\lambda}_9 = -\,q_9y_9 - r_2k_{I2}\{k_{P2}(\beta_2y_2 - A\,y_{12} - B\,y_{12} + A) + k_{I2}y_9\}$$

$$+ \frac{1}{T_{g2}}\,k_{I2}\,\lambda 4$$

$$\dot{\lambda}_{10} = 0$$

$$
\begin{aligned}
\dot{\lambda}_{11} = &- q_T A(A\, y_{12} + B\, y_{11} - A) - r_1 k_{P1} A\{k_{P1}(\beta_1 y_1 + A\, y_{12} \\
&+ B\, y_{11} - A) + k_{I1}\, y_8\} \\
&+ r_2 k_{P2} A\{k_{P2}(\beta_2 y_2 - A\, y_{12} - B\, y_{11} + A) + k_{I2} y_9\} \\
&+ \frac{A}{M_1}\lambda_1 - \frac{A}{M_2}\lambda_2 \\
&+ \frac{A}{T_{g1}} k_{P1}\lambda_3 - \frac{A}{T_{g2}} k_{P2}\lambda_4 - A\,\lambda_8 + A\,\lambda_9 - (y_2 - y_1)\lambda_{12}
\end{aligned}
$$

$$
\begin{aligned}
\dot{\lambda}_{12} = &- q_T B(A\, y_{11} + B\, y_{12} - A) - r_1 k_{P1} B\{k_{P1}(\beta_1 y_1 + A\, y_{12} \\
&+ B y_{11} - A) + k_{I1} y_8\} \\
&+ r_2 k_{P2} B\{k_{P2}(\beta_2 y_2 - A\, y_{12} - B\, y_{11} + A) + k_{I2} y_9\} \\
&+ \frac{B}{M_1}\lambda_1 - \frac{B}{M_2}\lambda_2 \\
&+ \frac{B}{T_{g1}} k_{P1}\lambda_3 - \frac{B}{T_{g2}} k_{P2}\lambda_4 - B\,\lambda_8 + B\,\lambda_9 - (y_1 - y_2)\lambda_{11}
\end{aligned}
$$

$$\dot{\lambda}_{13} = 0.0 \qquad (4.52)$$

and the gradient components are given by

$$H_{k_{P1}} = - r_1(\beta_1 y_1 + A\ y_{12} + B\ y_{11} - A)\ \{(\beta_1 y_1 + A\ y_{12} + B\ y_{11} - A)$$

$$+ k_{I1}\ y_8\} + \frac{1}{T_{g1}}\ (\beta_1 y_1 + A\ y_{12} + B\ y_{11} - A)\lambda_3$$

$$H_{k_{P2}} = - r_2(\beta_2 y_2 - A\ y_{12} - B\ y_{11} + A)\ \{(\beta_2 y_2 - A\ y_{12} - B\ y_{11} + A)$$

$$+ k_{I2} y_8\} + \frac{1}{T_{g2}}\ (\beta_2 y_2 - A\ y_{12} - B\ y_{11} + A)\lambda_4$$

$$H_{k_{I1}} = - r_1 y_8\{k_{P1}(\beta_1 y_1 + A\ y_{12} + B\ y_{11} - A) + k_{I1} y_8\} + \frac{1}{T_{g1}}\ y_8 \lambda_3$$

$$H_{k_{I2}} = - r_2 y_9\{k_{P2}(\beta_2 y_2 - A\ y_{12} - B\ y_{11} + A) + k_{I2}\ y_9\} + \frac{1}{T_{g2}}\ y_9\ \lambda_4$$

(4.53)

The non-intervention principle of che control actions was the strategy adopted in finding the optimal control parameters. It is assumed that both speed regulators and governors are operating simultaneously in response to the load changing in its own area.

First area 1, the steam area, was subjected to a different step load changing. It was assumed that the initial load angle difference between area 1 and area 2 was given as

$$\delta_{12}^{0} = 1.0 \text{ radian}$$

As was expected, when the system was subjected to small load disturbances, the optimal control parameters were found to be the same as in the case of the linear load frequency control problem. Table 1 shows the optimal control parameters; k_{P1} and k_{I1}, for different step load changing.

It can be seen from Table 3 that the optimal control parameters are not independent any more of the load changing and an adaptive controller is needed to get optimal performance for each load disturbance.

Second, area 2 the hydro area, was subjected to different step load changing. It was assumed that the initial operating condition characterized by the load angle difference between area 1 and area 2 was given as follows

$$\delta_{21}^{0} = 1.0 \text{ radian}$$

Similar results and conclusions were obtained as in the case of disturbing the steam area. These results are summarized in Table 4.

4.4.2 The effect of the initial power angle on the optimal control parameters

The conclusion that the areas' optimal control parameters depend on its area load changing was the motivation to study the effect of the initial power angle on the areas' control parameters.

Load Change	Initial k_{P1}	Initial k_{I1}	Initial Step Size	Optimal k_{P1}	Optimal k_{I1}	Norm-Square Gradient	Criterion
0.005	0.0	0.0	0.1	0.084	0.4577	0.211×10^{-5}	0.0239
0.01	0.0	0.0	0.1	0.085	0.464	0.295×10^{-5}	0.0943
0.02	0.0	0.0	0.01	0.09	0.475	0.691×10^{-5}	0.3681
0.03	0.0	0.0	0.01	0.095	0.485	0.134×10^{-5}	0.81
0.04	0.07	0.45	0.1	0.1	0.493	0.211×10^{-7}	1.41
0.05	0.07	0.45	0.1	0.109	0.499	0.656×10^{-4}	2.16
0.06	0.07	0.45	0.1	0.12	0.502	0.779×10^{-5}	3.06
0.07	0.07	0.45	0.1	0.129	0.502	0.316×10^{-5}	4.09
0.08	0.12	0.45	0.1	0.140	0.503	0.109×10^{-3}	5.26

Table 3

Steam area optimum control parameters versus area load disturbance

Load Change	Initial k_{P2}	Initial k_{I2}	Initial Step Size	Optimal k_{P2}	Optimal k_{I2}	Norm-Square Gradient	Criterion
0.005	0.0	0.0	0.1	0.665	0.325	0.58×10^{-5}	0.029
0.01	0.0	0.0	0.1	0.714	0.329	0.49×10^{-5}	0.114
0.02	0.0	0.0	0.1	0.793	0.334	0.42×10^{-5}	0.434
0.03	0.3	0.3	0.1	0.848	0.338	0.36×10^{-5}	0.941
0.04	0.7	0.3	0.1	0.887	0.34	0.41×10^{-5}	1.625
0.05	0.7	0.3	0.1	0.917	0.34	0.38×10^{-5}	2.481
0.06	0.7	0.3	0.01	0.938	0.34	0.30×10^{-5}	3.506
0.07	0.8	0.32	0.01	0.953	0.3397	0.31×10^{-5}	4.70
0.08	0.8	0.32	0.01	0.962	0.338	0.216×10^{-4}	6.06

Table 4

Hydro area optimum control parameters versus area load disturbance

δ°_{12} rad	0.4	0.6	0.8	1.0
k_{P1}	0.13	0.114	0.1	0.1
k_{I1}	0.636	0.600	0.563	0.499

Table 5

Steam area optimum control parameters

versus area initial power angle

δ°_{21} rad	0.3	0.5	0.7	0.9
k_{P2}	1.109	1.1025	1.059	0.976
k_{I2}	0.3193	0.321	0.32 7	0.337

Table 6

Hydro area optimum control parameters

versus area initial power angle

For example, if area 1 is being subjected to + 0.05 p.u. load disturbance, the relation between the initial load angle and the optimal control parameters is shown in Table 5. Similar results(shown in Table 6 are obtained when area 2 is being subjected to a 0.05 p.u. load disturbance.

It was noticed that in the case of neglecting the tie-line nonlinearity in the system dynamics that the system is always stable and can accommodate any amount of load disturbances. In contrast; in the case of taking the nonlinearity of the tie-line into consideration, there is a load increase limit for each power angle. For example, if δ_{12}° is equal to 1.0 rad, the maximum load which area 2 can fully accommodate is 0.025 p.u. In the case of δ_{12}° is equal to -1. radian, the maximum load increase which area 2 can withstand without losing its stability is 0.15 p.u.

In conclusion, the nonlinear LFC control parameters depend on the initial power angle and the megnitude of the load disturbance. However, the application of the optimal control parameters which were obtained from the linear analysis of the LFC to the nonlinear version of the problem results in an insignificant loss in the optimality performance index. For example, if area 1 is being subjected to a 0.06 p.u. load change and δ_{12}° is equal to 1.0, the value of the cost function in the case of employing the linear optimal control parameters to control the nonlinear system is

$$J_1 = 3.094$$

On the other hand, in the case of employing the nonlinear optimal control parameter, the value of the cost function is

$$J_2 = 3.053$$

The number

$$\varepsilon = \frac{J_1 - J_2}{J_1} \times 100$$

or

$$\varepsilon = 1.325 \text{ percent}$$

can serve as a suboptimality index for the nonlinear system under employing the linear optimal control parameters (suboptimal control parameters) to control the nonlinear system. Therefore, ε represents the price we have to pay, in terms of the performance index, to achieve better dynamic performance of the nonlinear system. However, the loss in the performance, in the case of employing the suboptimal control parameters instead of the optimal one is insignificant compared to the complications that arise in the design of the control system due to the necessity of the introduction of an observer in the system to identify the load change and consequently the corresponding optimal control parameters. Figure 11 shows a comparison between the system performance in the case of employing the optimal nonlinear control parameters and the linear one to control the nonlinear system.

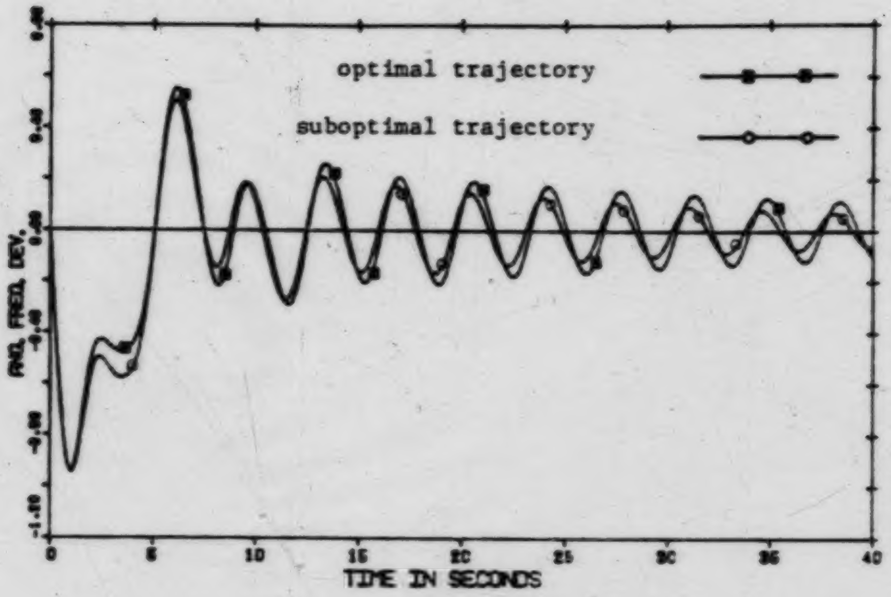

Fig. (11-a) Area 1 frequency deviation-time

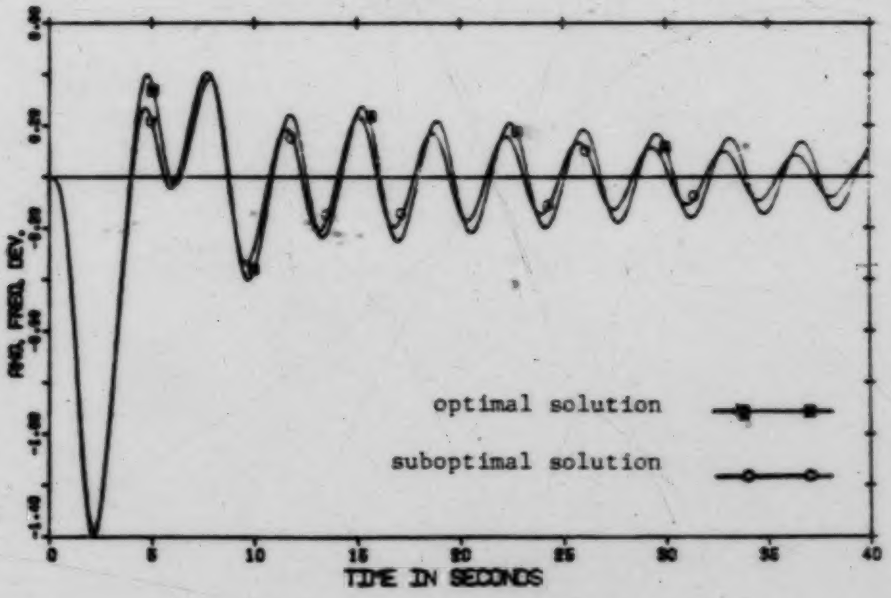

Fig. (11-b) Area 2 frequency deviation-time

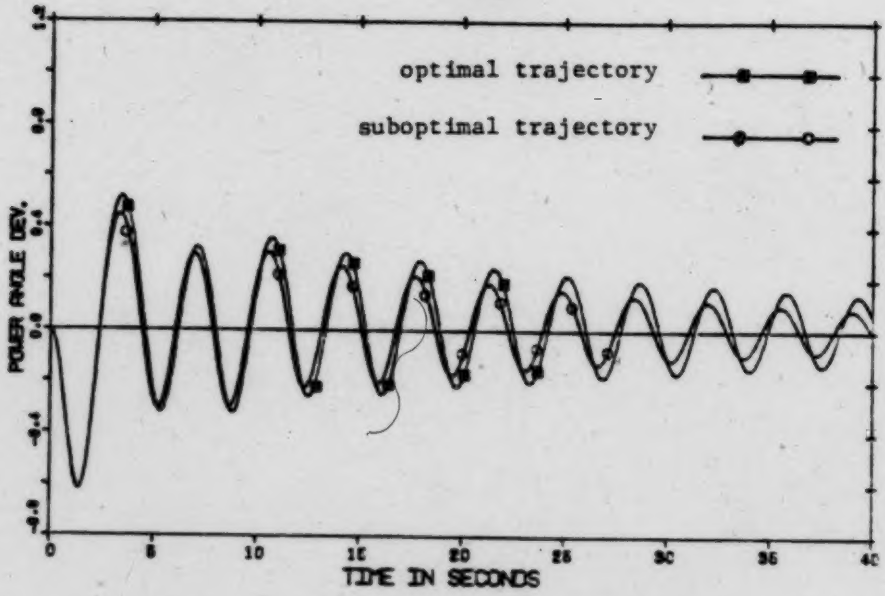

Fig. (11-c) Power angle deviation-time

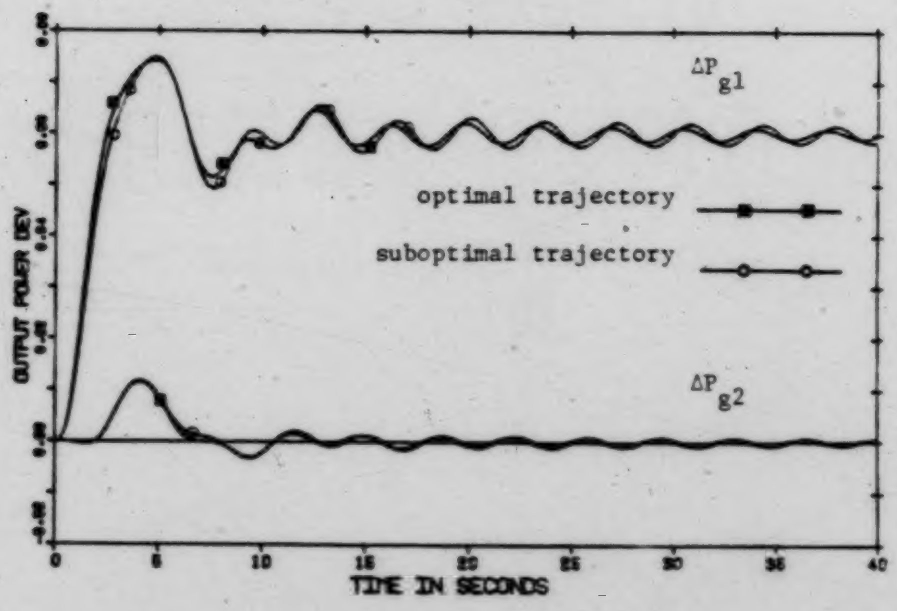

Fig. (11-d) Output power deviation-time

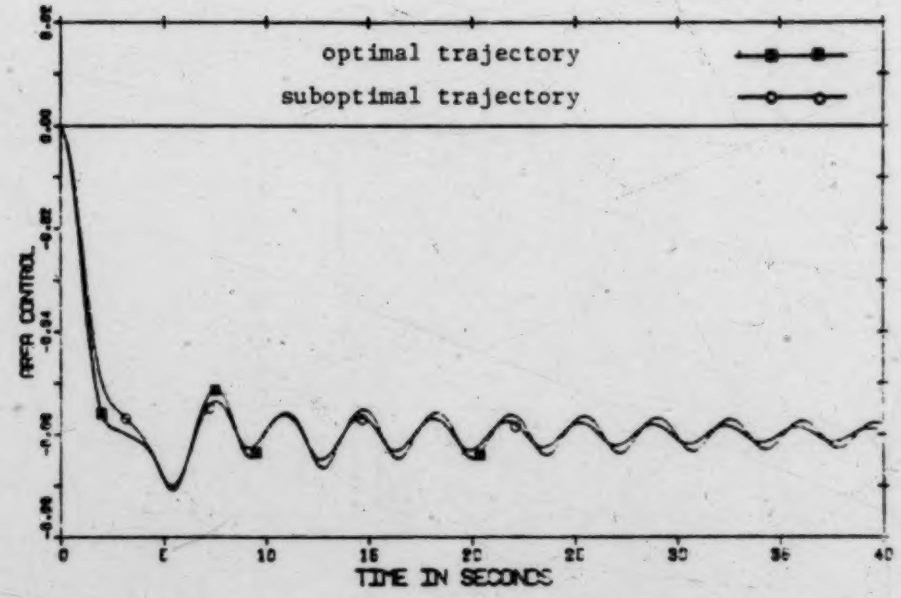

Fig. (11-e) Area 1 control-time

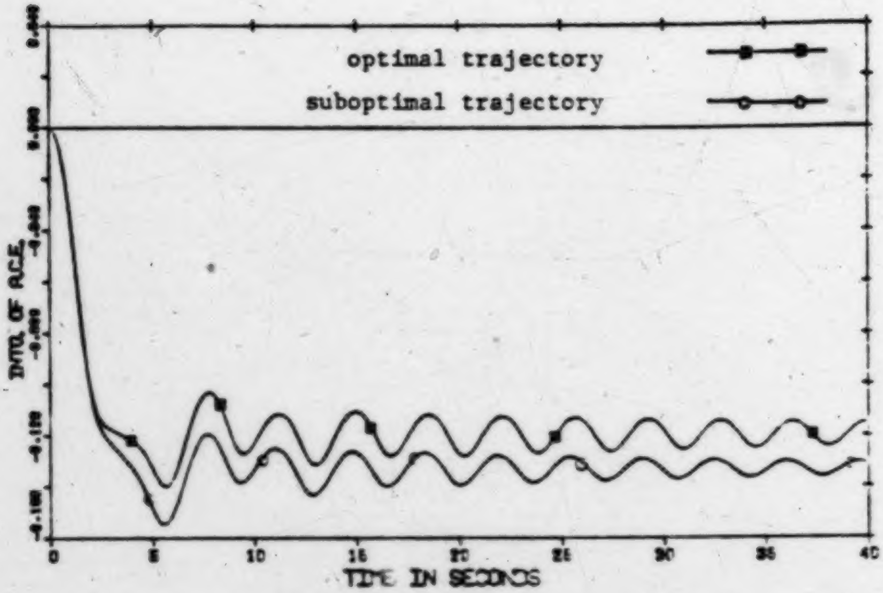

Fig. (11-f) $\int ace_1 dt$ - time

Comparison between system dynamic response of two area mixed system obtained from the nonlinear optimal controller parameters; optimal solution, and the linear optimal control parameters; the suboptimal solution.

Chapter 5

Load Frequency Control With Governor-Dead Band

5.1 Introduction

The governor-dead band is the hysteresis effect which arises from mechanical friction and/or looseness in the moving parts of the speed-governing system. That is, for any given controlled valve position, there may be a change in the control signal direction before the valve's position changes. Kirchmayer [6] has conducted an experimental analysis using an analog computer.to study the effect of the governor-dead band on the system behavior when the system is subjected to step load changes. It was shown that the dead-band effect is significant when the system is subjected to small load changes.

A governor hysteresis input-output characteristic may be described by the following equation

$$x_{out} = \begin{cases} x_{in} - \Delta & \dot{x}_{in} > 0 \\ x_{in} + \Delta & \dot{x}_{in} < 0 \\ \text{constant} & |x_{in} - x_{out}| \leq \Delta \end{cases} \qquad (5.1)$$

in which

2Δ = dead band

It is clear that hysteresis makes the system past history very important in influencing the system future response. This is in contrast to the systems considered in the preceeding two chapters, in which a previous state has no effect once the system trajectory has carried the system away from that state.

The problem of finding the optimal controls for a system which incorporates a hysteresis nonlinearity will be handled in the same manner in which systems without a hysteresis nonlinearity were handled in the preceeding chapters. The chief difference is that the adjoint system differs slightly from the one in which the hysteresis nonlinearity is not included. To show how to deal with systems having a hysteresis loop in an optimal sense, the problem of LFC of a single steam area will be considered.

5.2 Load frequency control of single steam area power system with dead band

A typical linearized model of a steam area power system with a governor dead-band is shown in Fig. (12) [14]. The control law, u, is specified to be proportional-plus-integral of the angular frequency deviation. In state space the system dynamics can be written as follows

$$\dot{x}_1 = -\frac{G}{M}x_1 + \frac{1}{M}x_4 - \frac{\Delta L}{M}$$

$$\dot{x}_2 = -\left([k_p + E]\frac{G}{M} + k_I\right)x_1 - \frac{k_p}{M}x_4 + k_p\frac{\Delta L}{M} + \frac{E\Delta L}{M} \qquad (5.2)$$

$$\dot{x}_3 = \frac{1}{T_g}x_{20} - \frac{1}{T_g}x_3$$

$$\dot{x}_4 = \frac{1}{T_t}x_3 - \frac{1}{T_t}x_4$$

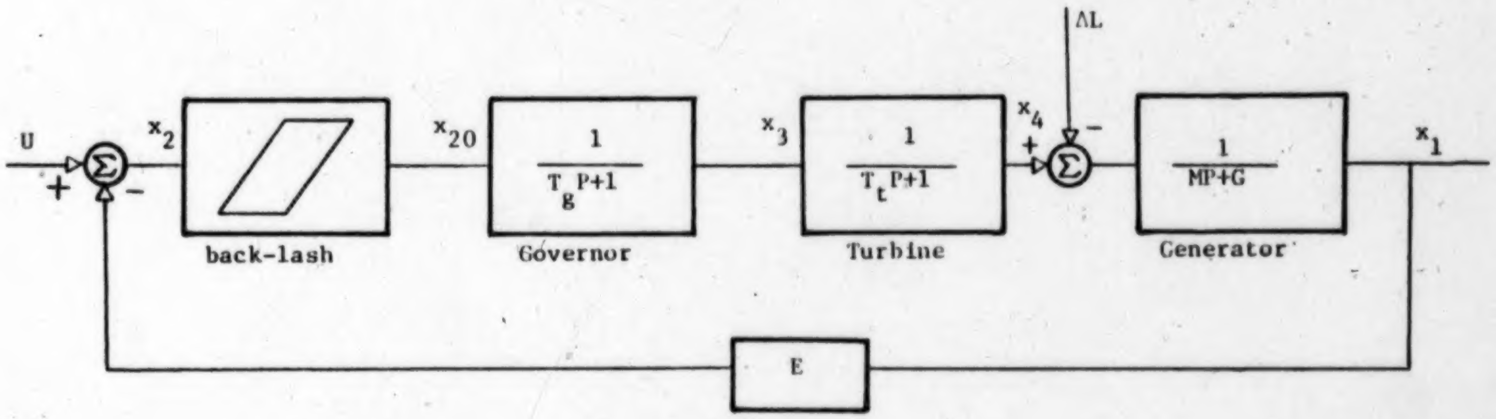

Fig. (12). Block diagram of a single steam area with dead-band

and the control law can be written as follows

$$u = - k_p x_1 - k_I \int_{t_0}^{t_f} x_1 dt \tag{5.3}$$

in which

$$x_{20} = \begin{cases} x_2 - \Delta & \dot{x}_2 < 0 \\ x_2 + \Delta & \dot{x}_2 > 0 \\ \text{constant} & |x_{20} - x_2| \leq \Delta \end{cases}$$

To cast the system dynamics into the familiar state space form, a new variable; x_5, is introduced

$$\dot{x}_5 = x_1 \tag{5.4}$$

and therefore the control law can be written as

$$u = - k_p x_1 - k_I x_5 \tag{5.5}$$

The problem posed is to find the optimal control parameters k_p and k_I, which minimize the cost functional

$$J = \frac{1}{2} \int_{t_0}^{t_f} \{ \sum_{i=1}^{5} q_i x_i^2 + r(k_p x_1 + k_I x_5)^2 \} dt \tag{5.6}$$

subject to the dynamic constraints given by equation (5.2). Here the rate generation constraint is not considered since the problem of LFC is studied for small load disturbances.

To handle a criterion of the form (5.6) a new state variable is defined

$$\dot{x}_6 = \frac{1}{2} \left(\sum_{i=1}^{5} q_i x_i^2 + r(k_p x_1 + k_I x_5)^2 \right) \qquad (5.7)$$

with

$$x_6(t_0) = 0$$

therefore, the cost functional can be written as follows

$$J = x_6(t_f) \qquad (5.8)$$

Now, by defining a Hamiltonian

$$H = \sum_{i=1}^{6} \lambda_i x_i \qquad (5.9)$$

then, by applying Pontryagin's minimum principle, one can get the system equations. The system state equations are given by equations (5.2), (5.4) and (5.7). The system co-state equations are found to be

$$\dot{\lambda}_1 = - q_1 x_1 + - r\, k_p (k_p x_1 + k_I x_5) + \frac{G}{M} \lambda_1 - ([k_p + E) \frac{G}{M} + k_I) \lambda_2$$

$$\dot{\lambda}_2 = - q_2 x_2 - S \frac{1}{T_g} \lambda_3$$

$$\dot{\lambda}_3 = - q_3 x_3 + \frac{1}{T_g} \lambda_3 - \frac{1}{T_t} \lambda_4 \tag{5.10a}$$

$$\dot{\lambda}_4 = - q_4 x_4 - \frac{1}{M} \lambda_1 + \frac{k_p}{M} \lambda_2 + \frac{1}{T_t} \lambda_4$$

$$\dot{\lambda}_5 = \qquad - r\, k_I (k_p x_1 + k_I x_5)$$

$$\dot{\lambda}_6 = 0$$

with

$$\lambda_i(t_f) = 0 \qquad i = 1,2,\ldots,6$$

in which

$$S = \begin{cases} 0 & \text{dead band is active} \\ 1 & \text{otherwise} \end{cases} \tag{5.10b}$$

the gradient vector components are given by

$$H_{k_p} = - r\, x_1 (k_p x_1 + k_I x_5) - (\frac{G}{M} x_1 - \frac{\Delta L}{M}) \lambda_2$$

$$H_{k_I} = - r\, x_5 (k_p x_1 + k_I x_5) - x_1 \lambda_2 \tag{5.11}$$

Since the integrand in the cost functional equation, equation (5.6), and the system dynamics are not explicit functions of t, we have

$$\dot{H} = 0$$

or

H = constant on the optimal trajectory

Therefore, in the dead-band zone the following condition

$$\frac{\partial H}{\partial x}\Big|_{t_d^-} = \frac{\partial H}{\partial x}\Big|_{t_d^+}$$

should be satisfied, that is,

$$\lambda_i(t_d^-) = \lambda_i(t_d^+) \quad i=1,2,\ldots,6 \qquad (5.12)$$

and

$$H_{k_p}(t_d^-) = H_{k_p}(t_d^+)$$

in which

$$H_{k_I}(t_d^-) = H_{k_I}(t_d^+)$$

$$t_d^- = \lim_{\varepsilon\to 0} t_d - \varepsilon$$

$$t_d^+ = \lim_{\varepsilon\to 0} t_d + \varepsilon$$

where

$$\dot{x}_2(t_d^-) = \dot{x}_2(t_d^+) = 0$$

that is, the time at which the dead band starts or ceases to be active. Equations (5.12) are similar to the Weierstrass-Erdman corner conditions. A corner point of $x(t)$, is a point at which the derivative is not uniquely defined, i.e., a point at which dx/dt poses a jump discontinuity.

5.3 The numerical solution of TPBVP of systems incorporating a back lash element

The problem of finding the optimal control parameters of a P-I controller of a dynamic system which includes a back lash element has been reduced to the problem of solving a nonlinear two-point boundary value problem with multiple corner points.

To solve a two-point boundary value problem with multiple corner points, the technique which has been introduced in Chapter 2 with a slight modification will be adopted This modification will take place in the forward path of integration to facilitate the detection of the corner points. As the integrating process in the forward direction proceeds, two values of each $x_2(t_j)$ and $\dot{x}_2(t_j)$ $j = 1-2, i-1$, are stored. When a new point $x_2(t_i)$ is being integrated, two problems have to be solved before proceeding with the integration process. These are:

1. detection of the mode changing which may be classified as follows:
   - i - rising mode of operation
   - ii - falling mode of operation
   - iii - dead band mode of operation
2. locating the exact moment of mode transition, i.e., the

moments of entering and leaving the dead band (corner points).

Detection of the change of the mode of operation can be easily done as follows

(i) in the case of entering the dead band mode, the sign of the prevailing $\dot{x}_2$ is opposite to the previous one; that is,

$$\mathrm{sign}\{\dot{x}_2(t_i)\ \dot{x}_2(t_{i-1})\} = -\ \mathrm{ve}$$

(ii) in the case of leaving the dead band, the following inequality should be satisfied:

$$|x_2(t_r) - x_2(t_i)| \geq \Delta$$

in which

$x_2(t_r)$ = the last reversing point

(iii) in the case of a rising mode of operation:

$$\mathrm{sign}\{\dot{x}_2(t_i)\} = +\ \mathrm{ve}$$

(iv) in the case of a falling mode of operation

$$\mathrm{sign}\{\dot{x}_2(t_i)\} = -\ \mathrm{ve}$$

The problem of finding the exact time of entering the dead band, reversing $\dot{x}_2(t_r)$, can be done by employing one of the interpolation techniques. If at the present point, t_i, of integration

$$\text{sign}\{\dot{x}_2(t_i)\ \dot{x}_2(t_{i-1}) = -\ \text{ve}$$

is satisfied, this means that the system has already passed the reversing point (t_r). To determine t_r, one can employ a Hermite third order interpolation polynomial to approximate $x_2(t)$ through $x_2(t_{i-2})$ and $x_2(t_{i-1})$. The third order Hermite polynomial is given by the following equation:

$$\hat{x}_2(t) = \sum_{j=i-2}^{i-1} x_2(t_j) H^*_{i-1,j}(t) + \sum_{j=i-2}^{i-1} \dot{x}_2(t_j)\ \hat{H}_{i-1,j}(t) \tag{5.13}$$

in which

$$H^*_{r-1,j} = \{1-2(t-t_j)L'_{i-1,j}(t_j)\}L^2_{r-1,j}(t)$$

$$\hat{H}_{r-1,j} = (t-t_j)L^2_{r-1,j}(t)$$

and

$$L_{n,k} = \prod_{\substack{j=0 \\ j\neq K}}^{i-1} \frac{(t-t_j)}{(t_k-t_j)}$$

where

$$L' = \frac{dL}{dt}$$

This reduces to

$$\hat{x}_2(t) = a\,t^3 + b\,t^2 + ct + d \tag{5.14}$$

in which

$$a = \frac{-2[x_2(t_{i-1}) - x_2(t_{i-2})] + (t_{i-1} - t_{i-2})[\dot{x}_2(t_{i-1}) + \dot{x}_2(t_{i-2})]}{(t_{i-1} - t_{i-2})^3}$$

$$b = \{3(t_{i-1} + t_{i-2})[x_2(t_{i-1}) - x_2(t_{i-2}) +$$

$$(t_{i-1} - t_{i-2})[(t_{i-1} + 2t_{i-2})\dot{x}_2(t_{i-1}) + 2t_{i-1} + t_{i-2})\dot{x}_2(t_{i-2})]\}$$

$$/(t_{i-1} - t_{i-2})^3$$

$$c = \{- G\, t_{i-1}t_{i-2}[x_2(t_{i-1}) - x_2(t_{i-2})] + (t_{i-1} - t_{i-2})$$

$$[(2t_{i-1}t_{i-2} + t_{i-1}^2)\dot{x}_2(t_{i-1}) + (t_{i-1}^2 + 2t_{i-1}\,t_{i-2})\dot{x}_2(t_{i-1})]\}$$

$$/(t_{i-1} - t_{i-2})^3$$

$$d = \{(3t_{i-1} - t_{i-2})t_{i-2}^{2}\ x_2(t_{i-1}) + (t_{i-1} - 3t_{i-2})t_{i-1}^{2}\ x_2(t_{i-2})$$

$$- (t_{i-1} - t_{i-2})t_{i-1}\ t_{i-2}[t_{i-2}\dot{x}_2(t_{i-1}) + t_{i-1}\dot{x}_2(t_{i-2})]\}$$

$$/\ (t_{i-1} - t_{i-2})^3$$

The reversing point (t_r) occurs when

$$\frac{d\hat{x}_2(t)}{dt} = 3a\ t_r^2 + 2bt_r + c = 0$$

or

$$t_r = -\frac{b}{3a} + \sqrt{\frac{b^2}{9a^2} - \frac{c}{3a}} \qquad (5.15)$$

Knowing the reversing time, the program re-integrates $x_2(t_r)$. If

$$|x_2(t_r) - \hat{x}_2| \leq \varepsilon$$

where ε is a predetermined limit

then $\hat{x}_2(t_r)$ or $x_2(t_r)$ is accepted as a corner point. If not, $\dot{x}_2(t_{i-1})$ will be replaced by $x_2(t_r)$ and the process is repeated until the reversing point is located accurately. Then $x_2(t_r)$ is stored.

The problem of finding the exact time of leaving or entering the dead band can be solved by employing an inverse interpolation process. Once again the Hermite third order polynomial will be used iteratively to find the exact point of entering or leaving the dead band. This can be done by solving the following equation

$$x_2(t_r) \pm 2\Delta = a\ t_d^3 + b\ t_d^2 + c\ t_d + d \qquad (5.16)$$

where t_d is the time at which the dead band starts to be inactive. Equation (5.16) can be solved analytically, but it has been found that the analytical solution accuracy is dependent on the coefficients a, b and c. The Newton Raphson method has been employed to solve equation (5.16) instead of the analytical method. As in the case of finding reversing points, the process of finding t_d will be iteratively repeated until t_d is determined accurately.

Having solved the problems of finding the reversing, entering, and leaving points, one can solve the two-point boundary value problem given by equation (5.2) and (5.10) using the algorithm of chapter 2 to get the optimal control parameters.

5.4 The optimal solution

The system was subjected to different load disturbances. It was found that the effect of the back lash element in the system dynamics on the optimal control gain parameters is overriding for small load disturbances. In the case that the system is being subjected to a load disturbance of magnitude somewhat greater than 0.05 p.u. for the following system parameters

$$T_g = 0.5, \quad T_t = 0.5, \quad M = 0.04, \quad G = 0.01, \quad \text{and} \quad E = 0.03$$

It was found that the optimal control gain parameters with and without taking into consideration the back lash effect are the same. For load disturbances of magnitude somewhat less than 0.05 p.u., the optimal control gain parameters depend on the disturbance magnitude. For example, if the system is being subjected to a load disturbance of - 0.004 p.u., the optimal control parameters, when the back lash element was taken into account in the system dynamics, were found to be

$$k_p = 0.0567$$

$$k_I = 0.0297$$

and the cost functional

$$J = 0.639 \times 10^{-2}$$

In the case of applying the optimal control gain parameters

$$k_p = 0.086$$

$$k_I = 0.022$$

which are obtained by neglecting the nonlinearity due to the back lash element (suboptimal control parameters) to the system with back lash, the cost functional was found to be

$$J_5 = 0.725 \times 10^{-2}$$

Therefore, by adopting a suboptimal strategy, there is a loss in the optimality performance index as follows

$$\varepsilon = \frac{(J_s - J_0)}{J_0} \times 100 = 13.45\%$$

A comparison between the system behavior in the two cases is shown in Fig. (13).

To show the effect of the back lash element (governor dead band) on the system behavior, the system was considered to be subjected to - 0.01 p.u., load change. Again, it was found that the optimal control parameters with the back lash element are different from those obtained by neglecting it. Also, it was found that the back lash element introduces a sustained oscillation in system behavior of approximately 0.2 Hz as well as increasing the maximum overshoots. A comparison between the system behavior in the two cases is shown in Figs. (14).

Conclusion

The problem of finding the optimal controls for a system which incorporates a back lash nonlinearity can be handled by employing Pontryagin's minimum principle and the Weierstrass-Erdmann corner conditions. The problem of detecting the corner points was solved by employing interpolation and extrapolation techniques. The remaining part of the problem is to solve the two-point boundary value problem. This was done by employing the conjugate-gradient-descent technique discussed in Chapter 2.

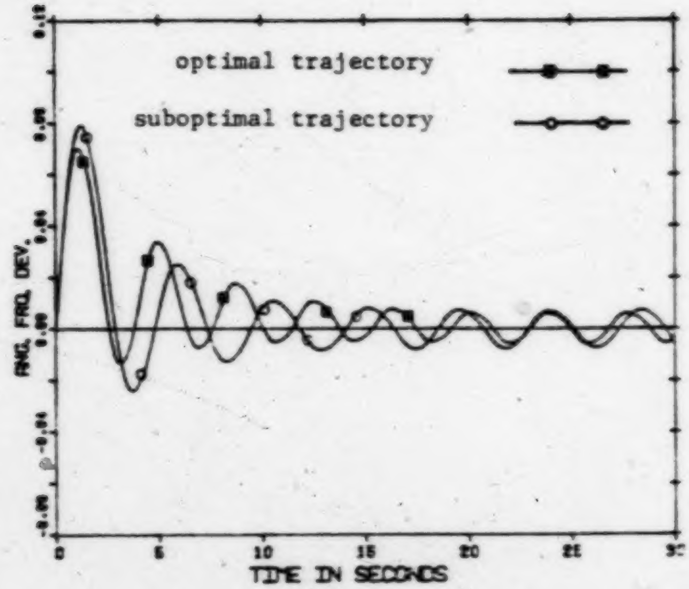

Fig. (13-a) Frequency deviation-time

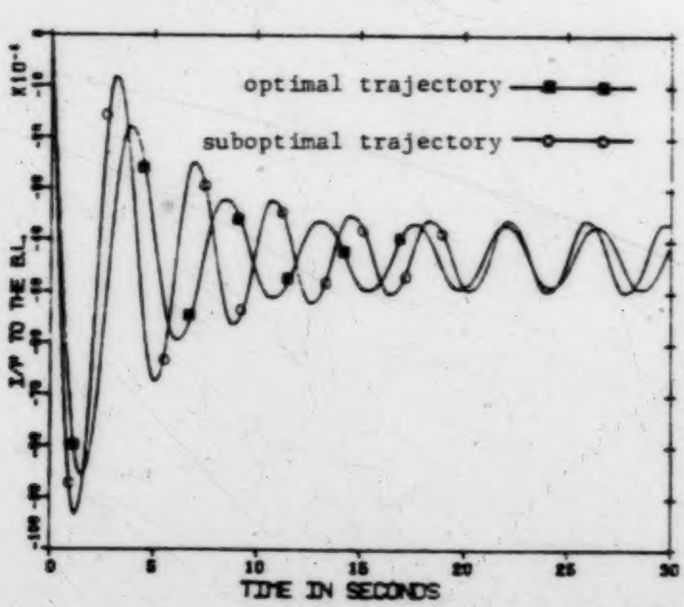

Fig. (13-b) Backlash input-time

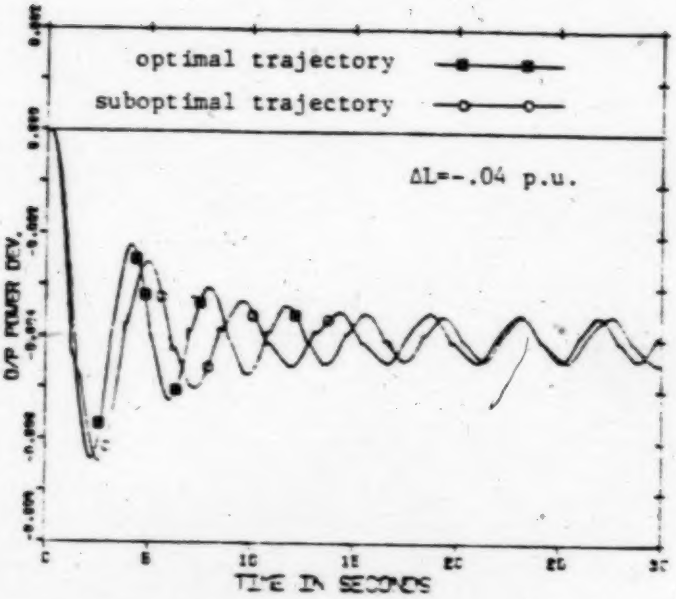

Fig. (13-c) Output power deviation-time Comparison between the system dynamic response of steam area system obtained from the nonlinear optimal controller; optimal solution, and the linear optimal control parameters; suboptimal solution

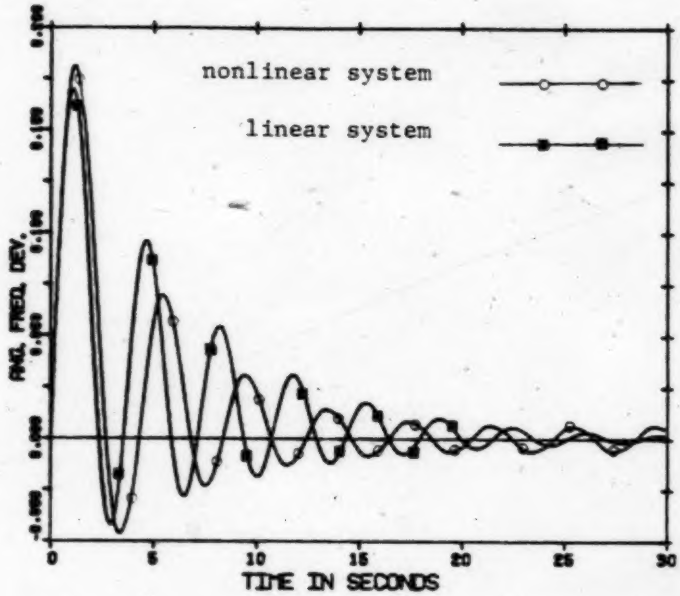

Fig. (14-a) Frequency deviation-time

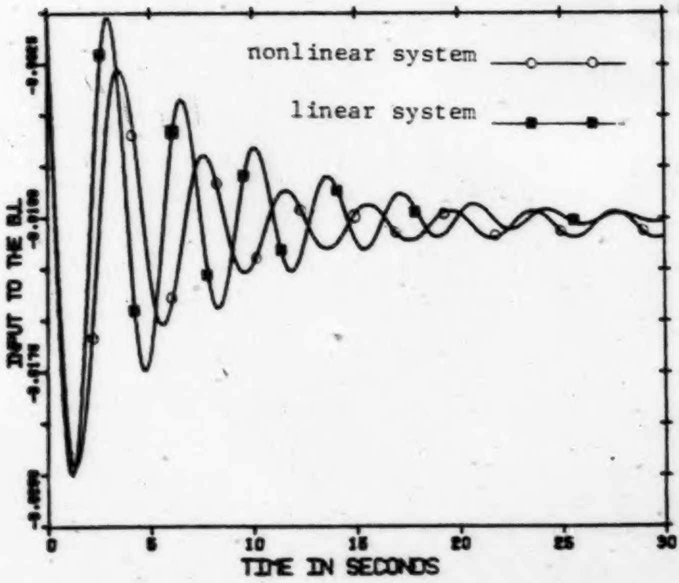

Fig. (14-b) Backlash input-time

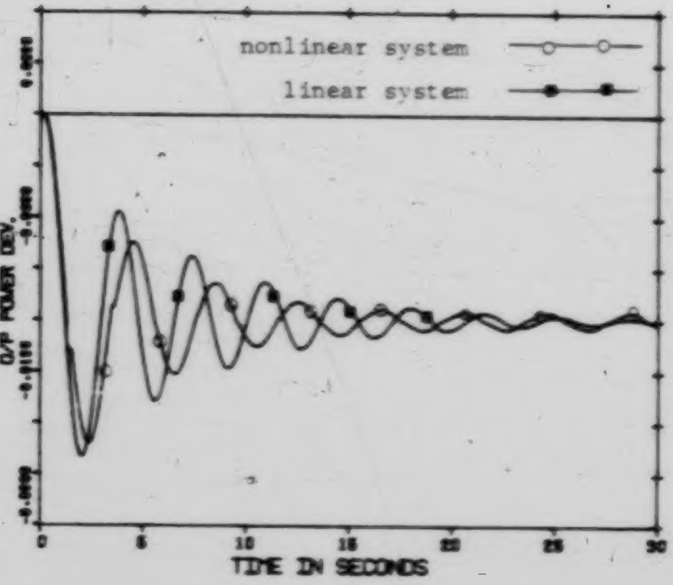

Fig. (14-c) Output power deviation-time

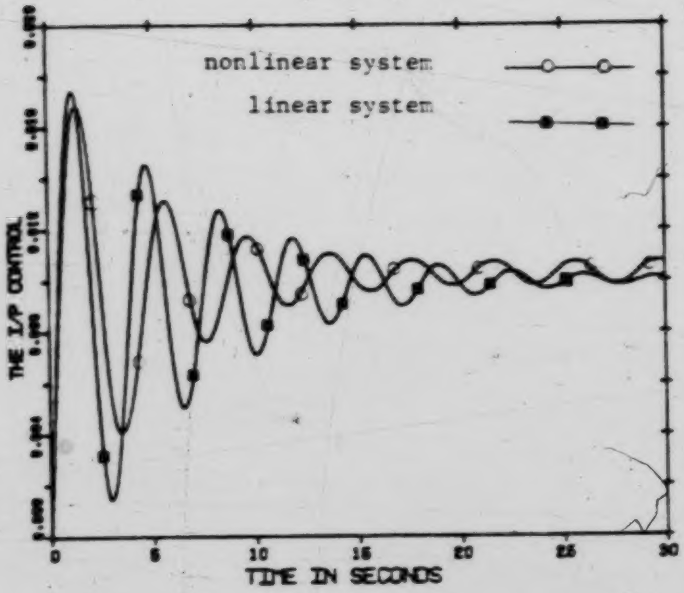

Fig. (14-d) Area control-time

Comparison between the system dynamic response of single steam area with and without backlash element; $\Delta L = -0.01$ p.u.

Chapter 6

Conclusions

A general numerical method for solving a deterministic parameter optimization problem using the steepest descent-conjugate-gradient algorithm is presented in Chapter 2. This general method can be implemented in large classes of optimal control design problems, especially those in which a solution of a nonlinear two-point boundary value problem is required to be found. A computer programme is presented in Appendix A which has been tested by available solved dynamic optimization problems [65].

The problem of load frequency control (LFC) of a single and multiple area interconnected power system was formulated as a parameter optimization one. An especially attractive feature of the proposed scheme is that it adopts present-day practice in the field of load frequency control and implements present-day-optimal control theory. The control law is specified to have proportional and/or integral action (the existing LFC strategy). By formulating the problem of LFC as one of parameter optimization allows us to compute the optimal control gain parameters off-line. This simplifies the on-line implementation of the proposed control strategy.

In Chapter 3, the problem of LFC of a single area steam and hydro power system is considered. The optimal control parameters in the case of employing an integral and/or proportional action are found to be independent of the load changing. In the case of a single steam area power system, a proportional-plus-integral-plus-derivative (PID) control action is employed to improve the system relative stability. Two schemes were adopted to find the optimal control parameters. In the first one, the optimal control parameters were found without any constraints on the pole locations except that they have to be in the L.H.S. of the s-plane. In the second one, the pole locations were preassigned in advance and the problem was converted to find the optimal derivative control parameter (suboptimal control parameters). In both cases, the optimal control parameters were found to be dependent on the load variations.

In spite of the fact that the optimal or the suboptimal PID controller gives better performance than the one resulting from the optimal integral and/or proportional control, it is recommended to employ the optimal PI controller to avoid the problem of designing an adaptive controller as well as the noise problem.

In Chapter 4, the problem of LFC of an interconnected power system is considered. First, the problem was solved for the case of neglecting the tie-line nonlinearity and the areas' governor deadband. In this case the optimal control parameters were found to be independent of the load changing. A sensitivity analysis was performed, and it was shown that the cost functional (the optimality index) is more sensitive to the areas' governor time constant than the rest of the areas' time constants. Therefore, it is recommended to put more emphasis on the process of identifying the areas' governor-time constant.

In the case of taking into account the tie-line nonlinearity, the optimal control parameters were found to be dependent on the areas' load variations. However, the loss in the system optimality when the linear optimal control parameters are employed to control the nonlinear system, is worthwhile in the sense that we avoid the problem of designing an adaptive controller which is required to estimate the areas' load disturbances and accordingly the proper control parameters.

In Chapter 5, the problem of taking into consideration a memory-type nonlinearity (hysteresis) in designing optimal control systems is considered. The problem of finding the optimal controls of a system which incorporates a hysteresis nonlinearity can be handled by employing the Pontryagin's minimum principle and the Weierstrass-Erdman corner conditions. The problem of detecting the corner points was solved by employing extrapolation and inverse interpolation Hermite techniques. The effect of the hysteresis element is prominent for small load disturbances. It introduces a sustained oscillation in the system response and it also increases the maximum overshoots.

Suggestions for further study

The foregoing work can be expanded to study the following:

(i) Valve and gate nonlinearities

In the case of a steam area system, the nonlinearity due to the turbine inlet valve arises from the relationship between the governor valve position x_v and steam flow q [66]. As the valve is progressively opened, the change in steam flow for a specified change in the valve position decreases. There is no available information in the literature about the relationship between x_v and q. Therefore, one should attempt to establish a basic model for this relationship. Then, further study could be conducted on the problem of LFC taking into account the valve nonlinearity employing the techniques developed in this thesis.

In a hydro area system, the relationship between the gate position and the turbine output power is not linear. In the case of a linear system it is given by

$$-\frac{Dp + 1}{0.5Dp+1}$$

where

$$P = \frac{d}{dt}$$

which is a linear approximation of [67]

$$-\frac{2\rho \tanh \; p\tau/2 + 1}{\rho \tanh \; p\tau/2 + 1}$$

in which

$$\tau\rho = L\overline{V}/g\overline{H} = D$$

where

D = water inertia constant

V = water speed

L = penstock length

H = head

g = specific gravity

A similar relationship can be found in [68].

(ii) Boiler dynamics

Further study can be done by taking into account boiler dynamics in the system dynamic equations [66].

APPENDIX A

This subroutine solves the problem of paramertric optimization using the steepest-descent-conjugate-gradient metods of minimization. All the instructions for how to use the code is documented in the subrotine grad. It is adivisable to test the code by one of the problems which has been discussed in this desertation.

```
C
      SUBROUTINE GRAD(YR,FR,YH,FH,TE,YPRED,DFV,SCV,DCV,CV,N1,NC)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 YR(N1),FR(N1),YH(4,N1),FH(3,N1),TE(N1),YPRED(N1),
     #DFV(NC),SCV(NC),DCV(NC),CV(NC),F
      INTEGER RUNGE
      COMMON Y1(151,20),Y2(151,20),Y3(6,20),DL1,DL2
      LOGICAL P
      DATA P/.TRUE./
      EXTERNAL DE1,DE2
C
C     REFERENCES
C     ----------
C     (1)-FOX,RICHARD "OPTIMIZATION METHODS FOR ENGINEERING DESIGN"
C         ADDISON-WESLEY COMPANY,1971
C     (2)-D.E.KIRK "OPTIMAL CONTROL THEORY:AN INTRODUCTION"
C         PRENTIC-HALL INC.,1970
C     (3)-FLETCHER, "PRACTICAL METHODES OF OPTIMIZATION"VOL.1
C         UNCONSTRAINED OPTIMIZATION,JOHN WILEY & SONS,1980
C     SUBROUTINES
C     -----------
C     THE USER SHOULD SUPPLY THE FOLLOWING SUBROUTINES
C
C     (1)- DE1   :STATE EQUATIONS
C     (2)- DE2   :CO-STATE EQUATINOS
C     (3)- SCINT :THE CO-STATE EQUATIONS COFFICIENTS WHICH ARE
C                 FUNCTIONS OF THE STATE EQUATIONS
C     (4)- TCOST :THE TERMINAL COST)
C     (5)- COSTAT:THE COSTATE EQUATIONS FINAL CONDITIONS
C
C     N=NO. OF THE STATE EQUATIONS
C     NC=NO. OF THE CONTROL PARAMETERS
C     N1=N+NC+1
C     NI=MAXIMUM NUMBER OF ITERATIONS
C     NT=MAXIMUM NUMBER TRIALS PER ITERATION
C     NR=NUMBER OF ITERATIONS PERFORMED BEFORE RESTSTING THE
C        CONJUGATE-GRADIENT METHOD(1 GIVES THE STEEPEST-DESCENT)
C     RATIO=MINIMUM VALUE OF ABS(D H(ALPHA)/D H(BETA))
C     DFMIN=MINIMUM VALUE OF ABS(THE GRADIENT OF COST FUNCTIONAL
C          W.R.T. THE CONTROL VECTOR)
C     FMIN=LOWER BOUND ON THE COST FUNCTIONAL
C     FMAX=UPPER BOUND ON THE COST FUNCTIONAL
C     ALMIN=MINIMUM INITIAL VALUE OF ALPHA
C     ALMAX=MAXIMUM INITIAL VALUE OF ALPHA
C     RMIN=MINIMUM FRACTION OF (ALPHA-BETA) USED IN INTERVAL
C          REDUCTION(<1)
C     RMAX=MAXIMUM FRACTION OF (ALPHA-BETA) USED IN
C          EXTERAPOLATION(>1)
C     XMAX=FINAL TIME
C     H=INTEGRATION STEP SIZE
C     F=J(C):THE VALUE OF THE COST FUNCTION
```

```
C     DFV(NC)= DJ(C)/DC
C
C     ARRANGE THE EQUATIONS IN DE1 AS FOLLOWS:
C         -THE STATE DYNAMIC EQUATIONS
C         -THE CONSTRAIN DYNAMIC EQUATIONS
C         -THE CONTROL DYNAMIC EQUATIONS
C         -THE COST FUNCTIONAL DYNAMIC EQUATION
C
C     ARRANGE THE EQUATIONS IN DE2 AS FOLLOWS:
C         -THE CO-STATE DYNAMIC EQUATIONS
C         -THE CO-CONSTRAIN DYNAMIC EQUATIONS
C         -THE CO-CONTROL DYNAMIC EQUATIONS
C         -THE CO-COST FUNCTIONAL DYNAMIC EQUATION
C
   10 FORMAT ('      XMAX,H')
   20 FORMAT (2D18.10)
   30 FORMAT (4D18.10)
   40 FORMAT (4X, 4D18.10)
   50 FORMAT (4I10)
   60 FORMAT ('1')
   70 FORMAT (//, '     NI,NT,NR,NC')
   80 FORMAT ('     RATIO,DFMIN,FMIN,FMAX')
   90 FORMAT ('     ALMIN,ALMAX,RMIN,RMAX')
  100 FORMAT ('     THE CONTROL VECTOR')
  110 FORMAT (//, '     ITERATION=', I2, '    TRIAL=', I2, '     F=',
     1       D18.10)
  120 FORMAT ('     ALPHA=', D18.10, '    DF=', D18.10, '    1DFV1=',
     1       D18.10)
  130 FORMAT (//, '        RESET THE CONTROL VECTOR')
  140 FORMAT (//, '        DESIGN COMPLETE')
  150 FORMAT (//, '        ITERATION LIMIT')
  160 FORMAT (//, '        TRIAL LIMIT')
C
      WRITE (6,60)
      READ (5,20) XMAX, H
      READ (5,50) NI, NT, NR, NC
      READ (5,30) RATIO, DFMIN, FMIN, FMAX
      READ (5,30) ALMIN, ALMAX, RMIN, RMAX
      WRITE (6,10)
      WRITE (6,20) XMAX, H
      WRITE (6,70)
      WRITE (6,50) NI, NT, NR, NC
      WRITE (6,80)
      WRITE (6,40) RATIO, DFMIN, FMIN, FMAX
      WRITE (6,90)
      WRITE (6,40) ALMIN, ALMAX, RMIN, RMAX
C
      N = N1 - NC - 1
      NIC = 0
      NTC = 0
      NRC = NR
C     ---------------
C     DC=(ALPHA-BETA)
C     ---------------
C     INITIALIZE THE CONTROL VECTOR
C     -----------------------------
      DO 170 J = 1, NC
  170 CV(J) = Y1(1,J + N)
C
      C = 0.D0
      X = 1.D0-1.D0 / RMAX
C     ---------------------------------------------------
C     INTEGRATE THE STATE EQUATIONS AND THE COST FUNCTION
C     DYNAMINC EQUATIONS FORWARD.
C     ---------------------------------------------------
      X1 = XMAX
      X2 = 0.D0
      I1 = (XMAX + .5*H) / H + 1
      CALL HAMING(DE1, YR, FR, YH, Y1, FH, TE, YPRED, N1, X1, H, X2, P)
C     -------------------------
C     COMPUTE THE TERMINAL COST
C     -------------------------
      DO 180 J = 1, N1
  180 YR(J) = Y1(I1,J)
      CALL TCOST(YR, FT)
      F = Y1(I1,N1) + FT
C     -----------------------------------------
C     INTEGRATE THE CO-STATE EQUATIONS BACKWARD
C     COMPUTE THE CO-STATE TERMINAL CONDITIONS
C     -----------------------------------------
      CALL COSTAT(YR)
C
      DO 190 J = 1, N1
  190 Y2(1,J) = YR(J)
C
      X1 = XMAX
C     ------------------------------------------------------
C     GENERATE THE SOLUTIONS FOR THE STATE EQUATIONS AT(XMAX,
C     XMAX-H/2,XMAX-H,...,XMAX-5H/2);USING RUNGE-KUTTA.
C     ------------------------------------------------------
      DO 200 J = 1, N1
        Y3(1,J) = Y1(I1,J)
  200 YR(J) = Y1(I1,J)
      I = 1
  210 K = RUNGE(N1,YR,FR,X1,-H/2.)
      IF (K .NE. 1) GO TO 220
      CALL DE1(YR, FR, X1)
      GO TO 210
  220 I = I + 1
      DO 230 J = 1, N1
  230 Y3(I,J) = YR(J)
      IF (I .LT. 6) GO TO 210
      X1 = 0.D0
      X2 = XMAX
      P = .TRUE.
C
      CALL HAMING(DE2, YR, FR, YH, Y2, FH, TE, YPRED, N1, X1, -H, X2, P)
C     --------------------------------------------------
C     COMPUTE THE GRADIENT OF THE COST FUNCTIONAL W.R.T.
C     CONTROL VECTOR.
C     --------------------------------------------------
      DO 240 J = 1, NC
  240 DFV(J) = Y2(I1,N + J)
C
      DF = 0.D0
      DO 250 J = 1, NC
        SCV(J) = CV(J)
        DCV(J) = -DFV(J)
  250 DF = DF + DCV(J) * DFV(J)
C
```

```
C      DF=-<DF(C)/DC,DF(C)/DC>
C
       SVP = -DF
       SDF = RATIO * DABS(DF)
       D3 = DSQRT(SVP)
       WRITE (6,110) NIC, NTC, F
       WRITE (6,120) C, DF, D3
       WRITE (6,100)
       WRITE (6,40) (CV(I),I=1,NC)
       IF (FMAX .GT. F) GO TO 260
       WRITE (6,130)
       RETURN
C      ------------------------------------------------------
C      CHECK THE TERMINATION OF THE TRIALS AND THE ITERATINS.
C      ------------------------------------------------------
   260 IF (DFMIN .LT. D3) GO TO 270
       WRITE (6,140)
       WRITE (6,60)
       WRITE (6,30) (CV(I),I=1,NC)
       RETURN
C      ----------------------------------------------------
C      INTIALIZE THE INCREMENT OF THE STEP SIZE DC USING THE
C      ONE-DIMENSIOL DIFFERENTIAL SEARCH ALGORITHM.
C      ----------------------------------------------------
   270 DC = 2.D0 * (FMIN - F) / DF
   280 DC = DMIN1(ALMIN,DMAX1(DC,ALMAX))
   290 NTC = NTC + 1
       A = C
       FA = F
       DFA = DF
       C = A + DC
       DO 300 I = 1, NC
   300 CV(I) = SCV(I) + C * DCV(I)
C      ------------------------------------------------------------
C      INTEGRATE THE STATE AND THE CO-STATE EQUATIONS WITH THE NEW
C      CONTROL VECTOR
C      ------------------------------------------------------------
       DO 310 J = 1, NC
   310 Y1(1,J + N) = CV(J)
       X1 = XMAX
       X2 = 0.D0
       P = .TRUE.
       CALL HAMING(DE1, YR, FR, YH, Y1, FH, TE, YPRED, N1, X1, H, X2, P)
       DO 320 J = 1, N1
   320 YR(J) = Y1(I1,J)
       CALL TCOST(YR, FT)
       F = Y1(I1,N1) + FT
       CALL COSTAT(YR)
       DO 330 J = 1, N1
   330 Y2(1,J) = YR(J)
       X1 = XMAX
       DO 340 J = 1, N1
         Y3(1,J) = Y1(I1,J)
   340 YR(J) = Y1(I1,J)
       I = 1
   350 K = RUNGE(N1,YR,FR,X1,-H/2.)
       IF (K .NE. 1) GO TO 360
       CALL DE1(YR, FR, X1)
       GO TO 350
   360 I = I + 1
```

```
      DO 370 J = 1, N1
  370 Y3(I,J) = YR(J)
      IF (I .LT. 6) GO TO 350
      X1 = 0.D0
      X2 = XMAX
      P = .TRUE.
      CALL HAMING(DE2, YR, FR, YH, Y2, FH, TE, YPRED, N1, X1, -H, X2, P)
      DO 380 J = 1, NC
  380 DFV(J) = Y2(I1,N + J)
C     --------------------------------------------------------------
C     CONTROL VECTOR(CV=CV+ALPHA*GRAD. OF THE COST FUNCTIO W.R.T CV)
C     --------------------------------------------------------------
      DF = 0.D0
      D1 = 0.D0
      DO 390 J = 1, NC
        DF = DF + DCV(J) * DFV(J)
  390 D1 = D1 + DFV(J) * DFV(J)
      D2 = DSQRT(D1)
C     --------------------------------------------------------------
C     CHECK FOR THE NORMAL TERMINATION OF THE TRIAL PROCEDURE:
C     <DIRECTION(I),-DJ(CV(I))>/<DIRECTION(I),-DJ(CV(I+1))>.LE.RATIO
C     --------------------------------------------------------------
      IF (SDF .GE. DABS(DF)) GO TO 630
      WRITE (6,110) NIC, NTC, F
      WRITE (6,120) C, DF, D2
      WRITE (6,100)
      WRITE (6,40) (CV(I),I=1,NC)
      IF (FMAX .GT. F) GO TO 420
  400 IF (NT .GT. NTC) GO TO 410
      GO TO 630
C     --------------------------------------------------------------
C     WHENEVER F(CV+STEP*DIRCETION(I)).GT.FMAX REDUCE STEP BY A
C     CERTAIN FACTOR RMIN IN THE RANGE OF (0,1) AND ADDITOINAL TRIAL
C     WILL BE PERFORMED
C     -------------------------------------------------------------
  410 DC = RMIN * DC
      C = A
      F = FA
      DF = DFA
      GO TO 290
C     ----------------------------------------------------
C     CHECK FOR <DIRECTION(I),F(CV+STEP*DIRECTION(I)>.GT.0
C     ----------------------------------------------------
  420 IF (DFMIN .LT. D2) GO TO 430
      WRITE (6,140)
      GO TO 660
  430 IF (DF) 440, 440, 470
  440 IF (NT .LT. NTC) GO TO 630
C     ----------------------------------------------------------
C     ESTIMATE THE OPTIMUM STEP SIZE BY QUADRATIC EXTERAPOLATION
C     ----------------------------------------------------------
      IF (X*DFA .LE. DF) GO TO 450
C     DC=DC*RMAX
C     GO TO 1021
C     -------------------------------------------------------
C     BY BASS THE QUADRATIC EXTRAPOLATION AND START NEW TRIAL
C     -------------------------------------------------------
      GO TO 630
  450 DC = DC * DFA / (DFA - DF)
      GO TO 290
  460 WRITE (6,160)
      GO TO 660
C     -------------------------------------------------------
C     ESTIMATE THE OPTIMUM STEP SIZE BY QUBIC INTTERAPOLATION
C     -------------------------------------------------------
  470 B = C
      FB = F
      DFB = DF
  480 D1 = DFA + DFB + 3.D0 * (FA - FB) / DC
      D2 = 2.D0 * (DFA + D1)
      D3 = DFA + DFB + 2.D0 * D1
      IF (1.D-5*DABS(D2) .LE. DABS(D3)) GO TO 490
      C = B - (DFA + 2.D0*D1) * DC / D2
      GO TO 500
  490 D2 = DSQRT(D1**2 - DFA*DFB)
      C = B - (DFB + D1 - D2) * DC / D3
  500 DO 510 I = 1, NC
  510 CV(I) = SCV(I) + C * DCV(I)
C     ------------------------------------------
C     INTEGRATE THE STATE AND CO-STATE EQUATIONS
C     ------------------------------------------
      DO 520 J = 1, NC
  520 Y1(1,J + N) = CV(J)
      X1 = XMAX
      X2 = 0.D0
      P = .TRUE.
      CALL HAMING(DE1, YR, FR, YH, Y1, FH, TE, YPRED, N1, X1, H, X2, P)
      DO 530 J = 1, N1
  530 YR(J) = Y1(I1,J)
      CALL TCOST(YR, FT)
      F = Y1(I1,N1) + FT
      CALL COSTAT(YR)
      DO 540 J = 1, N1
  540 Y2(1,J) = YR(J)
      X1 = XMAX
      DO 550 J = 1, N1
        Y3(1,J) = Y1(I1,J)
  550 YR(J) = Y1(I1,J)
      I = 1
  560 K = RUNGE(N1,YR,FR,X1,-H/2.)
      IF (K .NE. 1) GO TO 570
      CALL DE1(YR, FR, X1)
      GO TO 560
  570 I = I + 1
      DO 580 J = 1, N1
  580 Y3(I,J) = YR(J)
      IF (I .LT. 6) GO TO 560
      X1 = 0.D0
      X2 = XMAX
      P = .TRUE.
      CALL HAMING(DE2, YR, FR, YH, Y2, FH, TE, YPRED, N1, X1, -H, X2, P)
      DO 590 J = 1, NC
  590 DFV(J) = Y2(I1,N + J)
C
      DF = 0.D0
      D1 = 0.D0
      DO 600 I = 1, NC
        DF = DF + DCV(I) * DFV(I)
  600 D1 = D1 + DFV(I) * DFV(I)
      D2 = DSQRT(D1)
      IF (SDF .GE. DABS(DF)) GO TO 630
      NTC = NTC + 1
      WRITE (6,110) NIC, NTC, F
      WRITE (6,120) C, DF, D2
      WRITE (6,100)
      WRITE (6,40) (CV(I),I=1,NC)
      IF (FMAX .LE. F) GO TO 400
      IF (NT .LE. NTC) GO TO 630
      IF (DF) 620, 610, 610
  610 DC = C - A
      GO TO 470
  620 A = C
      FA = F
      DFA = DF
      DC = B - C
      GO TO 480
  630 NIC = NIC + 1
      DC = C
      C = 0.0D0
      NTC = 0
      IF (NRC .GT. NIC) GO TO 640
C     ----------------------------------------
C     RESTART THE CONJUGATE GRADINET PROCEDURE
C     ----------------------------------------
      SVP = 1.D50
      NRC = NRC + NR
  640 B = D1 / SVP
      SVP = D1
      DF = 0.D0
      DO 650 I = 1, NC
        SCV(I) = CV(I)
        DCV(I) = B * DCV(I) - DFV(I)
  650 DF = DF + DCV(I) * DFV(I)
      SDF = RATIO * DABS(DF)
      WRITE (6,110) NIC, NTC, F
      WRITE (6,120) C, DF, D2
      WRITE (6,100)
      WRITE (6,40) (CV(I),I=1,NC)
      IF (NI .GT. NIC) GO TO 280
      WRITE (6,150)
  660 WRITE (6,60) (SCV(I),I=1,NC)
      RETURN
      END
      FUNCTION RUNGE(N, Y, F, X, H)
C
C     The FUNCTION RUNGE EMPLOYS THE FOURTH-ORDER RUNGE-KUTTA
C     METHOD WITH CUTTA'S COEFFICIENTS TO INTEGRAT A SYSTEM OF
C     N-SIMULTANEOUS FIRST ORDER DIFFERENTIONAL EQUATIONS.
C     Maximun number of equations is equal to 50
C
      IMPLICIT REAL*8(A - H,O - Z)
      REAL*8 X, H
      INTEGER RUNGE
      REAL*8 PHI(50), SAVEY(50), Y(50), F(50)
      DATA M /0/
C
      M = M + 1
      GO TO (10, 20, 40, 60, 80), M
C
C     PASS 1
```

```
C     ------
C
   10 RUNGE = 1
      RETURN
C
C     PASS 2
C     ------
C
   20 DO 30 J = 1, N
        SAVEY(J) = Y(J)
        PHI(J),= F(J)
   30 Y(J) = SAVEY(J) + .5 * H * F(J)
      X = X + .5 * H
      RUNGE = 1
      RETURN
C
C     PASS 3
C     ------
C
   40 DO 50 J = 1, N
        PHI(J) = PHI(J) + 2. * F(J)
   50 Y(J) = SAVEY(J) + .5 * H * F(J)
      RUNGE = 1
      RETURN
C
C     PASS 4
C     ------
C
   60 DO 70 J = 1, N
        PHI(J) = PHI(J) + 2. * F(J)
   70 Y(J) = SAVEY(J) + H * F(J)
      X = X + .5 * H
      RUNGE = 1
      RETURN
C
C     PASS 5
C     ------
C
   80 DO 90 J = 1, N
   90 Y(J) = SAVEY(J) + (PHI(J) + F(J)) * H / 6.
      M = 0
      RUNGE = 0
      RETURN
C
      END
C
      FUNCTION HMING(N, Y, YPRED, F, X, H, TE, PRED)
C     -----------------------------
C     HMING IMPLEMENTS HAMMING'S PREDICTOR-CORRECTOR ALGORITHM
C     TO SOLVE N SIMLUTANEOUS FIRST-ORDER DIFFRENTIONALEQUATIONS.
C
      IMPLICIT REAL*8(A - H,O - Z)
      REAL*8 X, H
      INTEGER HMING
      LOGICAL PRED
      REAL*8 YPRED(N), TE(N), Y(4,N), F(3,N)
C
C     IS CALL FOR PREDICTOR OR CORRECTOR SECTION
C     ------------------------------------------
C
```

```
      IF ( .NOT. PRED) GO TO 40
C
C     PREDICTOR SECTION OF HMING
C     COMPUTE PREDICTED Y(J) VALUES AT NEXT STEP
C     ------------------------------------------
C
      DO 10 J = 1, N
   10 YPRED(J) = Y(4,J) + 4. * H * (2.*F(1,J) - F(2,J) + 2.*F(3,J)) / 3.
C
C     UPDATE THE Y AND F TABLES
C     -------------------------
      DO 20 J = 1, N
        DO 20 K = 1, 3
          L = 5 - K
          Y(L,J) = Y(L - 1,J)
   20 IF (L .LT. 4) F(L,J) = F(L - 1,J)
C
C     MODIFY PREDICTED Y(J) VALUES USING THE TRUNCATION ERROR
C     ESTIMATES FROM PREVIOUS STEP, INCREMENT X VALUE
C     ----------------------------------------------
C
      DO 30 J = 1, N
   30 Y(1,J) = YPRED(J) + 112. * TE(J) / 9.
      X = X + H
C
C     SET  PREDAND REQUEST UPDATE DERIVATIVE VALUES
C     ---------------------------------------------
C
      PRED = .FALSE.
      HMING = 1
      RETURN
C
C     CORRECTOR SECTION OF HMING
C     COMPUTE CORRECTED AND IMPROVED  VALUES OF THE Y(J) AND SAVE
C     TRUNCATION ERROR ESTIMATES FOR THE CURRENT STEP
C     -----------------------------------------------
C
   40 DO 50 J = 1, N
        Y(1,J) = (9.*Y(2,J) - Y(4,J) + 3.*H*(F(1,J) + 2.*F(2,J) - F(3,J)
     1  )) / 8.
        TE(J) = 9. * (Y(1,J) - YPRED(J)) / 121.
   50 Y(1,J) = Y(1,J) - TE(J)
C
C     SET PRED AND RETURN WITH THE SOLUTION FOR  THE  CURRENT STEP
C     ------------------------------------------------------------
C
      PRED = .TRUE.
      HMING = 2
      RETURN
C
      END
C
      SUBROUTINE HAMING(DE, YR, FR, YH, Y, F, TE, YPRED, N, XMAX, H, X,
     1             P)
C
      IMPLICIT REAL*8(A - H,O - Z)
      INTEGER RUNGE, HMING
      REAL*8 X, H, XMAX
      REAL*8 TE(N), YR(N), FR(N), Y(101,N), F(3,N), YPRED(N), YH(4,N)
      LOGICAL P
      EXTERNAL DE
C
C     Maximum no of output points is 101(it could changed into
C     any number by changing the dimension of matrix Y(*,N))
C
C     SET INITIAL TRUNCATION ERROR TO ZERO
C     SET THE FIRST ROW OF Y MATRIX
C
      I = 1
      DO 10 J = 1, N
        TE(J) = 0
        YH(4,J) = Y(I,J)
   10 YR(J) = Y(I,J)
C
C     CALL RUNGE TO INTEGERATE ACROSS THE FIRST THREE STEPS
C
   20 K1 = RUNGE(N,YR,FR,X,H)
      IF (K1 .NE. 1) GO TO 30
      CALL DE(YR, FR, X)
      GO TO 20
   30 I = I + 1
      DO 40 J = 1, N
        K = 5 - I
        YH(K,J) = YR(J)
   40 Y(I,J) = YR(J)
      CALL DE(YR, FR, X)
      DO 50 I1 = 1, N
   50 F(K,I1) = FR(I1)
      IF (I .LT. 4) GO TO 20
C
C     CALL HMING TO INTEGEATE ACROSS THE NEXT STEPS
C
   60 M = HMING(N,YH,YPRED,F,X,H,TE,P)
      DO 70 I2 = 1, N
   70 YR(I2) = YH(1,I2)
      CALL DE(YR, FR, X)
      DO 80 I3 = 1, N
   80 F(1,I3) = FR(I3)
      IF (H .LT. 0.D0 .AND. (X - XMAX) .LE. H/2.) GO TO 100
      IF (H .GT. 0.D0 .AND. (X - XMAX) .GE. H/2.) GO TO 100
      IF (M .EQ. 1) GO TO 60
      I = I + 1
      DO 90 J = 1, N
   90 Y(I,J) = YR(J)
      GO TO 60
C
  100 RETURN
      END
```

APPENDIX B

```
C
C       THIS SUBROUTINE SIMULATE THE BACKLASH ELEMENT DIGITIALLY
C
      SUBROUTINE BKLASH(YBLI, YPBLI, YBLO, X, H, DB, YI, YIP, YS, YSP,
     1          N, M, STATUS)
      IMPLICIT REAL*8(A - H,O - Z)
      REAL*8 Y(2), YP(2), YR(2), YI(N), YIP(N), YS(N), YSP(N)
      INTEGER STATUS
      COMMON /BLOCK3/ DUMMY, MODE(2), MODEDB
      COMMON /BLOCK4/ K(301)
      COMMON /BLOCK5/ J11
      LOGICAL START
C
C     Y(*)    WORK VECTOR OF LENGTH 2 TO STORE 2-VALUES OF THE INPUT
C             TO THE BACKLASH BEFORE REVERSING  THE DIRECTION
C     YP(*)   WORK VECTOR OF LENGTH 2 TO THE DERIVATIVE OF Y(*)
C             WITH RESPECT TO TIME
C     YI(*)   WORK VECTOR OF LENGTH N TO STORE THE VALUES OF THE
C             PREVAILING SYSTEM STATES AT X
C     YIP(*)  WORK VECTOR OF LENGTH N TO STORE THE VALUES OF THE
C             PREVALING SYSTEM  DERIVALTIVES AT X
C     YS(*)   WORK VECTOR OF LENGTH N  TO STORE THE VALUES OF THE
C             SYSTEM STATES AT X-H
C     YSP(*)  WORK VECTOR TO STORE THE DERIVATIVE OF YS(*)
C     YBLI    INPUT TO THE BACK-LASH
C     YPBLI   THE DERIVATIVE OF THE INPUT TO THE BACK-LASH
C     YBLO    OUTPUT FROM THE BACK-LASH
C     N       NUMBER OF STATE EQUATIONS
C     M       THE INDEX OF INPUT STATE TO THE BACKLASH ELEMENT
C     K(*)    VECTOR OF DIMENSION IMAX
C             K(*)=0 DEAD BAND IS ACTIVE
C             K(*)=1 DEAD BAND IS INACTIVE
C     MODE(2): PRESENT MODE OF OPERATION
C     MODE(1): PAST MODE OF OPERTAION
C               +1 RISING MODE
C               -1 FALLING MODE
C     MODEDB :1 DEAD BAND IS ACTIVE
C            :0 DEAD BAND IS INACTIVE
C     STATUS :2 INTERPOLATION OR EXTRAPOLATION MODE
C            :1 OTHERWISE
C     START=.TRUE.
C             MODE OF OPERATION IS AT THE HALF OF DEAD-BAND
C             (INITIAL CONDITION)
C     START=.FALSE.
C             MODE OF OPERATION WITH FULL DEAD-BAND
C     YR(*)   THE LAST TWO POINTS OF REVERSING DIRECTION
C     DB      DEAD BAND
C     J1=STATUS OF THE PREVAILING TWO POINTS
C     --------------------------------------
C     INITIALIZE THE SYSTEM MODE OF OPERATION
C     --------------------------------------
      STATUS = 1
C
      IF (X .GT. 1.1*H) GO TO 20
      START = .TRUE.
      MODEDB = 1
      XR = 0.0
C     -----------------
```

```
C     INITIALIZE Y,YP,YR,K
C     ------------------
      DO 10 I = 1, 2
        MODE(I) = 0
        Y(I) = 0.0
        YP(I) = 0.0
   10 YR(I) = 0.0
      K(1) = 0
C
      GO TO 40
C     -----------------------------------
C     CHECK UP FOR THE MODE OF OPERATION
C     -----------------------------------
   20 IF (YPELI) 70, 30, 110
   30 IF (YBLI .LT. 0 .AND. MODE(2) .LT. 0) YBLO = YBLI - DB / 2.
      IF (YBLI .LT. 0 .AND. MODE(2) .GT. 0) YBLO = YBLI + DB / 2.
      IF (YBLI .GT. 0 .AND. MODE(2) .LT. 0) YBLO = YBLI - DB / 2.
      IF (YBLI .GT. 0 .AND. MODE(2) .GT. 0) YBLO = YBLI + DB / 2
      MODEDE = 1
      GO TO 220
C     ------------------------------------------------------
C     AT THE FIRST STEP OF INTEGERATION (X.EQ.H) THE MODE
C     ASSME THAT AT X=H THAT YBLO=0.0 OR YBLI+-BD/2.
C     ------------------------------------------------------
   40 IF (YPELI .LT. 0) MODE(2) = -1
      IF (YPBLI .GT. 0) MODE(2) = 1
      IF (DABS(YBLI) .LE. DB/2.) GO TO 50
      IF (YBLI .LT. 0) YBLO = YBLI + DB / 2
      IF (YELI .GT. 0) YBLO = YBLI - DE / 2.
      MODEDE = 0
      START = .FALSE.
      GO TO 60
   50 YBLO = 0.0
   60 GO TO 220
C     --------------------------
C     FALLING MODE OF OPERATION
C     --------------------------
   70 MODE(1) = MODE(2)
      MODE(2) = -1
      J1 = MODE(1) * MODE(2)
      IF (MODEDB .EQ. 1) GO TO 180
      IF (J1 .LT. 0) GO TO 140
      IF ( .NOT. START) GO TO 90
      IF (DABS(YBLI) - DB/2.) 80, 80, 90
   80 YBLO = 0.0D0
      GO TO 220
   90 IF (YBLI .GT. 0) YBLO = YBLI - DB / 2.
      IF (YBLI .LT. 0) YBLO = YBLI + DB / 2.
  100 FORMAT (2X, E14.5, 2I3, 4E14.5/)
      GO TO 220
C     -------------------------
C     RISING MODE OF OPERATION
C     -------------------------
  110 MODE(1) = MODE(2)
      MODE(2) = 1
      J1 = MODE(1) * MODE(2)
      IF (MODEDB .EQ. 1) GO TO 180
      IF (J1 .LT. 1) GO TO 140
      IF ( .NOT. START) GO TO 130
      IF (DABS(YBLI) - DB/2.) 120, 120, 130
  120 YBLO = 0.000
      GO TO 220
  130 IF (YBLI .LT. 0) YBLO = YBLI + DB / 2.
      IF (YBLI .GT. 0) YBLO = YBLI - DB / 2.
      GO TO 220
C
  140 YR(1) = YR(2)
C     ---------------------------------------------------
C     IMPEMENT THE THIRD-ORDER HERMITE INTERPOLATION FORMULA
C     TO FIND THE REVERSING POINTS
C     ---------------------------------------------------
      CALL INTR(Y, YP, YS, YSP, X, H, YR(2), XDB, XR, N, M)
      STATUS = 2
C
C     CHECK FOR THE MODE OF OPERATION
C
      IF (DABS(YS(M) - YR(2)) .LT. DB) MODEDB = 1
      IF (DABS(YS(M) - YR(2)) .GE. DB) MODEDB = 0
C
      YP(1) = YP(2)
      Y(1) = Y(2)
      YP(2) = YSP(M)
      Y(2) = YS(2)
C
      IF ( .NOT. START) GO TO 150
      IF (START .AND. DABS(YR(2) - YR(1)) .LE. DB/2.) GO TO 160
  150 IF (MODE(1) .GT. 0 .AND. YR(2) .GT. 0.) YBLO = YR(2) - DB / 2.
      IF (MODE(1) .GT. 0 .AND. YR(2) .LT. 0.) YBLO = YR(2) + DB / 2.
      IF (MODE(1) .LT. 0 .AND. YR(2) .GT. 0.) YBLO = YR(2) - DB / 2
      IF (MODE(1) .LT. 0 .AND. YR(2) .LT. 0.) YBLO = YR(2) + DB / 2.
      MODE(1) = MODE(2)
      IF (YP(2) .LT. 0) MODE(2) = -1
      IF (YP(2) .GT. 0) MODE(2) = 1
      IF (DABS(YR(2) - YR(1)) .LT. DB) GO TO 250
C
  160 START = .FALSE.
C     WRITE(6,750) XR
  170 FORMAT (2X, 'REVERSING POINT AT XR=', E16.5/)
C     WRITE(6,400)X,MODE(1),MODE(2),YP(1),YP(2),YBLI,YBLO
      K(J11) = MODEDE - 1
      YBLOS = YBLO
      RETURN
C     ------------------------------------------
C     DETERMINE THE POINTS OF LEAVING THE DEAD-BAND
C     ------------------------------------------
  180 IF ( .NOT. START) GO TO 190
      IF (DABS(YBLI) .GT. DB/2.) GO TO 200
      GO TO 220
C
  190 IF (J1 .LT. 0) GO TO 140
C
      IF (MODE(1) .EQ. 1 .AND. YBLI .GT. (YR(2) + DB)) GO TO 200
      IF (MODE(1) .EQ. - 1 .AND. YBLI .LT. (YR(2) - DB)) GO TO 200
      GO TO 220
C
  200 MODEDE = 0
      CALL DDBAND(Y, YP, YS, YSP, X, H, YR(2), XDB, DB, N, M, START)
C     WRITE(6.23)XDB
  210 FORMAT (2X, 'DEAD-BAND LEAVING POINT=', E16.4/)
C     ------------------------------------------------------
```

```
C     CHECK FOR REFLECTION POINT JUST AFTER LEAVING THE DEAD-BAND
C     -----------------------------------------------------------
      STATUS = 2
C
      MODE(1) = MODE(2)
      IF (YSP(M) .GT. O) MODE(2) = 1
      IF (YSP(M) .LT. O) MODE(2) = -1
      J1 = MODE(1) * MODE(2)
      IF (J1 .LT. O) GO TO 140
      YP(1) = YP(2)
      YP(2) = YSP(M)
      Y(1) = Y(2)
      Y(2) = YS(M)
      IF (YS(M) .LT. O .AND. MODE(2) .GT. O) YBLO = YS(M) + DB / 2
      IF (YS(M) .GT. O .AND. MODE(2) .LT. O) YBLO = YS(M) - DB / 2
      IF (YS(M) .LT. O .AND. MODE(2) .LT. O) YBLO = YS(M) + DB / 2
      IF (YS(M) .GT. O .AND. MODE(2) .GT. O) YBLO = YS(M) - DB / 2
C     WRITE(6,400)X,MODE(1),MODE(2),YP(1),YP(2),YBLI,YBLO
      K(J11) = MODEDE - 1
      YBLOS = YBLO
C
      RETURN
C
  220 Y(1) = Y(2)
      Y(2) = YBLI
      YP(1) = YP(2)
      YP(2) = YPBLI
C     WRITE(6,400)X,MODE(1),MODE(2),YP(1),YP(2),YBLI,YBLO
  230 DO 240 I = 1, N
        YSP(I) = VIP(I)
  240 YS(I) = VI(I)
C
      STATUS = 1
      K(J11) = MODEDE - 1
      YBLOS = YBLO
      RETURN
C     -------------------------------------
C     CHANGE IN THE INPUT WITIN THE DEAD-BAND
C     -------------------------------------
  250 YBLO = YBLOS
      MODEDE = 1
C     WRITE(6,200)
  260 FORMAT (/'CHANGE IN THE INPUT WITHIN THE DEAD-BAND'/)
C     WRITE(6,400)X,MODE(1),MODE(2),YP(1),YP(2),YBLI,YBLO
      K(J11) = MODEDB - 1
      RETURN
      END
C
      SUBROUTINE DDBAND(Y, YP, YS, FS, X, H, YR, XDB, DB, N, M, START)
      IMPLICIT REAL*8(A - H,O - Z)
      REAL*8 Y(2), YP(2), YS(N), FS(N)
      INTEGER RUNGE1
      COMMON /BLOCK3/ YR1, MODE(2), MODEDB
      LOGICAL START
C
      YR1 = YR
      X1 = X - 2. * H
      X2 = X - H
      IF ( .NOT. START) GO TO 10
      IF (Y(2) .LE. 0.0) CO = YR - DB / 2.
      IF (Y(2) .GE. 0.0) CO = YR + DB / 2.
      START = .FALSE.
      GO TO 20
   10 IF (YR .LT. 0. .AND. MODE(2) .GT. 0) CO = YR + DB
      IF (YR .LT. 0. .AND. MODE(2) .LT. 0) CO = YR - DB
      IF (YR .GT. 0. .AND. MODE(2) .GT. 0) CO = YR + DB
      IF (YR .GT. 0. .AND. MODE(2) .LT. 0) CO = YR - DB
C
C     ----------------------------------------------------------
C     PERFORM INVERSE INTERPOLATION ;TO GET THE TIME AT WHICH
C     THE INPUT IS LEAVING THE DEAD BAND,USING NEWTONS FORMULA
C     (SECOND-ORDER)
C     ----------------------------------------------------------
C
   20 XDB = X1 + .1 * H
C
      J = 1
   30 A = 2 * (Y(1) - Y(2)) / (X2 - X1) ** 3 + (YP(2) + YP(1)) / (X2 -
     1X1) ** 2
      B = (3.*(X2 + X1)*(Y(2) - Y(1))) / (X2 - X1) ** 3 - ((2.*X2 + X1)*
     1YP(1) + (2.*X1 + X2)*YP(2)) / (X2 - X1) ** 2
      C = (6.*X2*X1*(Y(1) - Y(2))) / (X2 - X1) ** 3 + ((X2**2 + 2.*X2*
     1X1)*YP(1) + (X1**2 + 2.*X2*X1)*YP(2)) / (X2 - X1) ** 2
      D = ((X2 - 3.*X1)*X2**2*Y(1) + (3.*X2 - X1)*X1**2*Y(2)) / (X2 -
     1X1) ** 3 - (X2*X1*(X2*YP(1) + X1*YP(2))) / (X2 - X1) ** 2 - CO
C
C     SOLVE AX**3+BX**2+CX+D USING NEWTON'S METHOD
C     THE ANALYTICAL SOLUTION OF THE QUBIC EQUATION CAN BE
C     DEDUCED BUT THE ACCURACY OF ITS NUMERICAL SOLUTION IS
C     VERY POOR
C
      K = 1
   40 FSS = A * XDB ** 3 + B * XDB ** 2 + C * XDB + D
      DFSS = 3.D0 * A * XDB ** 2 + 2.D0 * B * XDB + C
      IF (DABS(FSS) - .1D-5) 60, 60, 50
   50 XDB = XDE - FSS / DFSS
      K = K + 1
      IF (K .GT. 30) GO TO 60
      GO TO 40
C
   60 IF (XDB .LT. X .OR. XDB .GT. X2) GO TO 80
      WRITE (6,70)
   70 FORMAT ('THERE IS NO ROOT IN DDBAND'/)
      STOP
C
C     DETERMINE BACK LASH LEAVING POINT
C     INTEGERAT USING RUNGE-KUTTA METOHED.
C
   80 H1 = XDB - X2
      XRUNGE = X2
   90 K1 = RUNGE1(N,YS,FS,XRUNGE,H1)
      IF (K1 .NE. 1) GO TO 100
      CALL DE11(YS, FS, XRUNGE)
      GO TO 90
  100 CALL DE11(YS, FS, XDB)
C
C     CONTINUE INTEGRATION FROM XDB TO X
C
      MODEDB = 0
      H1 = X - XDB
  110 K1 = RUNGE1(N,YS,FS,XDB,H1)
      IF (K1 .NE. 1) GO TO 120
      CALL DE11(YS, FS, XDB)
      GO TO 110
  120 CALL DE11(YS, FS, X)
      RETURN
      END
C
      SUBROUTINE INTR(Y, YP, YS, FS, X, H, YR, XDB, XR, N, M)
      IMPLICIT REAL*8(A - H,O - Z)
      REAL*8 Y(2), YP(2), YS(N), FS(N)
      INTEGER RUNGE1
      COMMON /BLOCK3/ YR1, MODE(2), MODEDB
C
C     ------------------------------------------------------------
C     FIND THE REVERSING TIME BY IMPLEMENTING HERMITE INVERSE
C     INTERPOLATION FORMULA (QUBIC-INTERPOLATION)
C     ------------------------------------------------------------
C
      J = 1
      YR1 = YR
      X1 = X - 2. * H
      X2 = X - H
      XR = (X1 + X2) / 2
C
   10 A = (-6.*(Y(2) - Y(1)) + 3.*(X2 - X1)*(YP(2) + YP(1))) / (X2 - X1)
     1 ** 3
C
      B = (6.*(X2 + X1)*(Y(2) - Y(1)) - 2.*(X2 - X1)*((X2 + 2.*X1)*YP(2)
     1 + (2.*X2 + X1)*YP(1))) / (X2 - X1) ** 3
C
      C = (-6.*X2*X1*(Y(2) - Y(1)) + (X2 - X1)*((2.*X2*X1 + X1**2)*YP(2)
     1 + (X2**2 + 2.*X2*X1)*YP(1))) / (X2 - X1) ** 3
C
      K = 1
   20 FSS = A * XR ** 2 + B * XR + C
      DFSS = 2. * A * XR + B
      IF (DABS(FSS) - .1D-5) 40, 40, 30
   30 XR = XR - FSS / DFSS
      K = K + 1
      IF (K .GT. 30) GO TO 40
C
   40 IF (XR .LT. X .OR. XR .GT. X2) GO TO 60
      WRITE (6,50)
   50 FORMAT ('THERE IS NO ROOT IN INTR'/)
      STOP
C
C     INTEGRATE THE SYSTEM'S DYNAMICS FROM X2 TO XR
C
   60 H1 = XR - X2
      XRUNGE = X2
   70 K1 = RUNGE1(N,YS,FS,XRUNGE,H1)
      IF (K1 .NE. 1) GO TO 80
      CALL DE11(YS, FS, XRUNGE)
      GO TO 70
   80 CALL DE11(YS, FS, XR)
      X2 = XR
      YR = YS(M)
C
C    CONTNUE INTEGRATION FRO XR TO X
```

```
C
      STATUS = 1.
      MODEDB = 1
      H1 = X - XR
   90 K1 = RUNGE1(N,YS,FS,XR,H1)
      IF (K1 .NE. 1) GO TO 100
      CALL DE11(YS, FS, XR)
      GO TO 90
  100 CALL DE11(YS, FS, X)
C
      RETURN
      END
```

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